**UNIVERSIDAD NACIONAL DE INGENIERIA**
**FACULTAD DE CIENCIAS**

# TESIS
# DIRAC NEUTRINOS IN AN $SU(2)$ LEFT-RIGHT SYMMETRIC MODEL

PARA OBTENER EL GRADO ACADÉMICO DE DOCTOR EN CIENCIAS CON MENCIÓN EN FÍSICA

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# Contents

# *Abstract*

It is known in the Standard Model of particle physics that the parity symmetry is violated (left-right asymmetry of fermions) in the weak interaction, consequently, neutrinos are considered as particles not massive, however, according to experimental facts, like in the case of neutrino oscillation, they must have mass. For this reason, one way to correct this asymmetry is to extend the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ to a group where the left and right fields are transformed in the same way, these models are called left-right symmetric models, where due to the presence of the right fields of the neutrinos these can gain mass.

This work propose a left-right symmetric model, where the scalar sector consisting of two doublets and two bidoublets (could include more bidoublets), in which neutrinos remain as Dirac fermions, like the charge leptons and quarks, in all orders in perturbation theory. However, only with two bidoublets the neutrino masses still need a fine-tuning, this would not be the case when a third bidoublet is added to our scalar sector. One of the scalar doublets may be regarded as inert, because the left-right symmetry forbids it to couple with the known fermions[1].

[1]H. Diaz, V. Pleitez and O. Pereyra Ravinez. Dirac neutrinos in a SU(2) left right symmetric model. Phys. Rev. D102, 075006 (2020)

# Introduction

With the advancement of technology applied to the construction of new detectors, and the effort of many dedicated experimentalists[1], we now know that neutrinos appear in nature in three different flavors where each type of neutrino is grouped with its charge lepton partern. Moreover, the different observations, where involving neutrinos, have played a key role in our understanding about weak interactions in the building of the Standard Model (SM) of the particle physics[2] and it is believed that further observations may be the cornerstone to understand the physics beyond the SM.

The known electroweak theory (also known as the GlashowWeinbergSalam theory) is a unified theory describing the weak and electromagnetic interaction of elementary particles. Its prototype was Glashow's model to combine the weak and electromagnetic interaction in the framework of $SU(2)_L \otimes U(1)_Y$ symmetry. Weinberg and Salam supplemented the Higgs mechanism to generate masses of gauge particles and fermions, and succeeded in placing the model in the mathematical framework of gauge theories.

The experimental evidence shows that neutrinos are characterized by having half-integer spin, in others words they are fermions, and have no electric charge[1][3]. Having a little mass or not was one of the key questions of the particle physics and constituted one of the chief obsessions in the scientific thought; however due to the evidence of neutrino oscillation, they must have mass, small but they do.

The SM, like a model at low energies, predicts massless neutrinos, making them different from other fermions such as the charged leptons (e, $\mu$, $\tau$) and the quarks ($u$, $d$, $s$, $c$, $t$, $b$), which introducing the spontaneous symmetry breaking and the Higgs mechanism these known fermions acquire mass. As is knowing the charge leptons and quarks presents left and right fields within the framework of the model. In addition, the SM has also been extremely successful in explaining the various low weak energy processes involving charged and neutral current interactions of neutrinos[3]. Many enteresenting models which go beyond the SM[1] that became extremely popular in fact were closely tied to the masslessness of the neutrino[2].

Among the extended models which may manifest itself in the multi-TeV (or maybe higher) range of energies are the left-right symmetric models[4][5] in the electroweak sector. In these models, the left and right chiralities of leptons and quarks are assumed to play an identical role previous to the symmetry breaking (or at high energies above all symmetry breaking scales). Furthermore, it follows that in the left right symmetric models, weak interactions conserve parity[4, 6], being this property already shared by strong, electromagnetic and gravitational interactions. Maybe, this is closer to the essential part of unified theories than the SM. The weak interactions treatment in the left right symmetric models requires that all left handed fermions must have a right handed partner. Therefore, an immediate consequence of these models is the existence of a new particle, the right handed neutrino, usually denoted by $\nu_R$ (lighter neutrino) or sometimes by $N_R$ (heavy neutrinos are predicted by others left right models where the neutrino is the Majorana type). In conclusion, these models propose a complete correspondence between leptons and quarks in its spectrum and leads us to obtain massive neutrinos[6], which could be either Majorana or Dirac ones.

As mentioned in previous paragraphs, the simplest extension of the SM involving additional charged gauge bosons is the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ model, in which the first (second) $SU(2)$ couples to L (R) chiral fermions. In the original versions of the model[5][7], the generator of the $U(1)$ symmetry is indeed $B-L$. It was appreciated subsequently [6], where the electric charge is expressed as a lineal combination of the generators of the symmetry groups: $Q = I_{3L} + I_{3R} + \frac{B-L}{2}$ (Gell Mann - Nishijima formula).

In the first chapter, we will talk about some basic points about the SM of the particle physics, considering the particles that it contains and how they are classified, as well as the process of mass generation through the Higgs mechanism[8] and will discuss some alternatives of getting massive neutrinos, which represents extended models respect to the SM.

In the second chapter, we will discuss about the theoretical structure of our model with gauge symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes \mathcal{P}$, which represents one of the extended models of the SM, a model with right-handed fermions (we will have right-handed neutrinos) that presents the same law of transformation as the already known left-handed fermions of the SM, taking into account a scalar sector that must respect all known symmetries (Lorentz invariant, gauge invariant, and invariant with respect to some discrete symmetries). With the $\mathcal{P}$ operator, this left-right symmetric model respects the parity symmetry from the beginning[4]. After spontaneous symmetry breaking and using the Higgs mechanism[8], these particles proposed by the model acquire mass, including neutrinos. we also study the different sectors proposed by the model, for example, in the gauge sector we will see the way as the gauge bosons interact with the leptons. I can mention that this thesis work has two published articles, one of them in the Physical Review D whose title is Dirac neutrinos in an SU(2) left-right symmetric model and the other one published in the Journal of Physics G, titled Explicit parity violation in $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$.

In the third chapter, we develop some phenomenological calculations; firstly, in the scalar sector through the more general potential where applying certain discrete symmetries we get a simpler version than the first one. With this later version of the scalar potential we obtain the new constrain equations. In addition, we work with the gauge sector where using the covariant derivatives related to the scalar sector we can obtain the masses and the physical states of the vector bosons. Finally, we obtain the coupling coefficients between the leptons and the physical gauge bosons. In addition, the couplings between quarks with the neutral vector bosons have also been obtained.

In the fourth chapter, We work with the yukawa sector where, by choosing a parameterized mixing matrix (PMNS), we obtain for our model the Yukawa couplings for both charged leptons and neutrinos, using information obtained from the particle data group (PDG)[3], that is, it has been regarded different types of hierarchy for the neutrinos masses like: Normal hierarchy, Inverted hierarchy and the Cuasi-degenerated hierarchy without phase factor.

In the fifth chapter some of the phenomenological consequences due to the characteristics of the model are discussed, where it is commented on the existence of FCNCs in the scalar sector, as well as on contributions to the magnetic moment of the muon, ways to recognize if neutrinos are of Dirac or Majorana type through the performance of certain processes. A brief discussion of the electric dipole moment of the fundamental particles is also treated.

In the sixth chapter, the conclusions are established according to what was gotten in the present work, considering that the objectives have been achieved and are shown in the articles published in the journals mentioned in the previous paragraphs.

# Chapter 1

# The Standard Model of Particle Physics

## 1.1 Introduction of the Electroweak Standard Model (ESM)

The success of the ESM predictions is remarkably high and even, if we can extent it beyond what one would have expected. It shows a very elegant theoretical framework, which since about 1974 has successfully explained our observations at particle colliders. The main parts of the SM were introduced by Salam and Weinberg independently around 1968, however Weinbergs paper for example received only two citations in 1969 and 1970.

The ESM,[1][2] describe only three of the known fundamental forces (the electromagnetic, weak, and strong interactions, without including the gravitational one) in the universe, moreover it explains how the all known elementary particles are classified.

This model is built as a result of several experiments and symmetry principles like Lorentz invariance, local gauge invariance and electric charge conservation. Moreover, after of the discovery of the Higgs boson at LHC [9], the SM received the status of Physical phenomena theory in the electroweak energy scale (up to about 200 GeV).

The principles of gauge symmetry represent one important part the great success of ESM as it establishes an connection between local (gauge) symmetries and forces mediated by spin-1 particles called vector gauge bosons. On the other hand, in the ESM, the weak and electromagnetic interactions are connected to gauge symmetry under the direct product $SU(2)_L \otimes U(1)_Y$ where $L$ for left-handedness, and $Y$ for hypercharge ( the fields' charge before spontaneous symmetry breaking). The ESM gauge symmetry is spontaneously broken to $U(1)_Q$ (natural symmetry) where couples to the electromagnetic charge $Q = T_{L3} + Y$ which can be obtained after applying the spontaneous symmetry breaking (the Gell Mann- Nishijima relation). $T_{L3}$ represents the weak isospin which is the third generator of $SU(2)_L$. The model explains the interactions of the known fermions (only electroweak sector) once they are assigned to well defined representation of the gauge group.[1][10].

It is important to keep in mind that in all gauge theories the particles masses are protected by the gauge group of the model, the symmetry group of the ESM fixes the interactions, that is, the number and properties of the vector gauge bosons. In addition, the two coupling constants $g$ and $g_Y$ of the $SU(2)_L$ and $U(1)_Y$ groups respectively must be determined from experiments. On the other hand, the number and properties of scalar bosons and fermions are left unconstrained, except for the fact that they must be transformed in a particular way under the symmetry group.

From an experimental and theoretical point of view there is too much information, already known in the literature, about the success of the SM, however, we are interested in how the fermions (quarks, charged leptons) acquire mass. Take into account that in the SM formulation the neutrinos, after the breakdown of symmetry, remain as non-massive. This is what will be emphasized in the following sections of this chapter.

## 1.2 Quarks and Charge Leptons masses in the SM

In the formulation of the SM, the fermions masses are generated via Yukawa coupling, where the only scalar Higgs doublet, $\Phi$, couples with the fermions proposed by the SM, that is, a right-handed component must couple with the left-handed one. The former is an $SU(2)_L$ doublet, the latter is part of a singlet.

$$-\mathcal{L}_{Yukawa} = Y^d_{ij}\,\overline{Q}'_{Li}\,\Phi\,D'_{Rj} + Y^u_{ij}\,\overline{Q}'_{Li}\,\tilde{\Phi}\,U'_{Rj} + Y^\ell_{ij}\,\overline{\ell}'_{Li}\,\Phi\,\ell'_{Rj} + H.C., \tag{1.1}$$

where $\tilde{\Phi} = i\,\tau_2\Phi^*$ which after spontaneous symmetry breaking these terms lead to charged fermion masses, that is[1]:

$$m^f_{ij} = \frac{v}{\sqrt{2}}\,Y^f_{ij}, \tag{1.2}$$

$v$ is the vacuum expectation value (V.E.V.) of the Higgs field. Nevertheless, since the model does not contain right-handed neutrinos, the Yukawa interaction of Eq. (1.1) leave the neutrinos massless.

### 1.2.1 Neutrinos in the ESM: Massless Neutrinos

In the ESM, we know neutrinos are fermions don't feel neither strong nor electromagnetic interactions. In others words, they are singlets under $SU(3)_C$ and their hypercharge is $-1/2$. They transform as isospin doublets of $SU(2)_L$:

$$\Psi'_{L\ell} = \begin{pmatrix} \nu'_\ell \\ \ell' \end{pmatrix}_L$$

where, in general, $\psi'_L\,(=\ell'_L, \nu'_{\ell L})$ is the left-handed component of the fermion $\psi$:

$$\psi'_L = P_L\psi' \equiv \frac{1}{2}\,(1-\gamma_5)\,\psi'$$

. On the other hand, in what follows we can define the **active neutrinos** to those which form part of these lepton doublets. In the SM there is one active neutrino for each charged leptons, $e$, $\mu$, and $\tau$. $SU(2)_L$ gauge invariance dictates the form of weak charged current ($CC$) interactions between the neutrinos and their corresponding charged leptons and neutral current ($NC$) among themselves to be:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\sum_\ell \overline{\nu}_{L\ell}\,\gamma^\mu\,\ell^-_L\,W^+_\mu + H.C., \tag{1.3}$$

$$\mathcal{L}_{NC} = -\frac{g}{2\cos\theta_W}\sum_\ell \overline{\nu}_{L\ell}\,\gamma^\mu\,\nu_{L\nu}\,Z^0_\mu \tag{1.4}$$

where $\theta_W$ is the Weinberg angle and $g$ is the coupling constant associated to $SU(2)$ group, as was mentioned in the previous parragraph. In addition, the equations (1.3) and (1.4) describe all interactions in the ESM that neutrinos present. Taking into account the equation (1.4), it determines the decay width of the $Z^0_\mu$ boson into light left-handed neutrinos mass eigenstates. Then according to experimental evidences, the total decay witdh of the $Z^0_\mu$ boson, the number of such neutrinos (the particle data group [3] is $N_\nu = 2.984 \pm 0.008$, this value represent the number of light neutrinos which are coupled to $Z^0$, in others words this allows determining the number of species of light neutrinos. Whatever model representing an extension of the SM should contain three and only three light active neutrinos.

In models which are extensions of the SM, certain types of neutrinos called **Sterile Neutrinos** (or right-handed neutrinos) are proposed. These kinds neutrinos are defined as having no SM gauge interactions, their only interactions is gravitational, in others words, they are singlets of the complete SM gauge group and have hypercharge $Y = 0$. Thus the SM, as a gauge theory is able to describe all known particle interactions, leaving out sterile neutrinos[11].

¿Could a neutrino mass term be generated at loop level?, knowing the particle content of SM the only way to build neutrino mass term is through the bilinear term $\overline{\Psi}_L\, \Psi^c_L$, where $\Psi^c_L$ is the charge conjugated field, $\Psi^c_L = C\overline{\Psi}^T_L$ and $C$ is the charge conjugation operator. Nevertheles, this term is forbidden in the SM because it violates the total lepton symmetry by two units and hence it cannot be induced by loop corrections[12].

It is concluded that within the ESM neutrinos are precisely massless and the symmetry group needs to be extended to generate mass to neutrino.

## 1.3 Non-zero neutrino mass: Extensions of ESM

With the content of matter particle and gauge symmetry of the ESM we can conclude that the generation of mass terms for the neutrinos won't be posible. Therefore, in order to introduce a neutrino mass in the theory, it is necessary to extend the symmetry group of the model.

We could explore several posibilities to generate a neutrino mass term, without changing the gauge symmetry of the model, adding to the ESM an arbitrary number of sterile neutrinos $\nu_{sk}$ $(k = 1, 2, \cdots m)$. With the addition of $m$ number of sterile neutrinos (Extension of the ESM) one may build two gauge invariant renormalizable operators. Now there are, in general, two types of mass terms[1]:

$$-\mathcal{L}^{M_\nu} = M_{D_{ij}} \overline{\psi}'_{\nu,si} \psi'_{\nu,Lj} + \frac{1}{2} M_{N_{ij}} \overline{\psi}'_{\nu,si} \psi'^{c_{sj}}_{\nu} + H.C., \tag{1.5}$$

$\psi'^c = C\gamma^0\psi'^*$, is the charge conjugated field of the neutrino, moreover, $M_D$ is complex matrix of $(m \times 3)$ dimension and $M_N$ is a $m \times m$ matrix.

The Yukawa interactions after applying the spontaneous symmetry breaking can generate mass terms:

$$\Gamma^\nu_{ij} \overline{\psi}'_{si,\nu}\, \tilde{\phi}^\dagger L'_{Lj} \;\rightarrow\; \mathrm{M}_{D_{ij}} = \Gamma^\nu_{ij} \frac{v}{\sqrt{2}}, \tag{1.6}$$

similar to Eqs. (1.1) and (1.2) for the fermions in the ESM, where it is called a Dirac mass term and preserve the lepton number (total).

[1] Standard Model: A Primer-Burgess Moore

The Majorana mass term is the second in the equation (1.5), it differs from the Dirac mass terms in many relevant aspects:

1. It represents a singlet field of the ESM gauge group.
2. It breaks the lepton number in two unities. Therefore, such terms are not allowed for any charged fermions which, by definition, carry $U(1)_Q$ charges.

We can rewrite Eq. (1.5) at the form:

$$-\mathcal{L}_{M_\nu} \equiv \frac{1}{2}\overline{\vec{\psi}_\nu^{\prime c}} M_\nu \vec{\psi}_\nu^{\prime} + H.C., \tag{1.7}$$

where $\vec{\psi}_\nu^{\prime} = \left(\vec{\psi}_{\nu L}^{\prime},\ \vec{\psi}_{\nu s}^{\prime c}\right)^T$ is a vector with dimension $(3+m)$. In addition, $M_\nu$ (complex and symmetric matrix) can be diagonalized by a unitary matrix $V^\nu$ of dimension $(3+m)$, that is:

$$\hat{M}_\nu = (V^\nu)^T M_\nu V^\nu, \quad m_k : \text{real values}, k : 1, 2, \cdots m+3$$

The original weak eigenstates in terms of the resulting $3+m$ mass eigenstates are given by the following form:

$$\vec{\psi}_\nu = (V^\nu)^\dagger\ \vec{\psi}_\nu^{\prime}$$

Writing the Eq. (1.7) in terms of the mass eigenstates, we obtain:

$$-\mathcal{L}_{M_\nu} = \frac{1}{2}\sum_{i=1}^{k} m_i\ \left(\overline{\psi}_{i,\nu}^{c}\,\psi_{i,\nu} + \overline{\psi}_{i,\nu}\,\psi_{i,\nu}^{c}\right) \ = \ \frac{1}{2}\sum_{i=1}^{k} m_i\,\overline{\psi}_{Mi,\nu}\,\psi_{Mi,\nu}, \tag{1.8}$$

$k = m+3$, where:

$$\psi_{Mi,\nu} = \psi_{i,\nu} + \psi_{i,\nu}^{c} = \left(V^{\nu\dagger}\vec{\psi}_\nu\right)_i + \left(V^{\nu\dagger}\vec{\psi}_\nu\right)_i^c. \tag{1.9}$$

From this last expression we can observe the following relation ( called Majorana condition[2]):

$$\psi_{M,\nu} = \psi_{M,\nu}^{c} \tag{1.10}$$

These are the Majorana neutrinos where both neutrino and antineutrino states are described by only one field, on the other hand, for the case of charged fermion the particle and antiparticle are described by two different fields. Then, a Majorana like neutrino are described by spinor with two components, however, the charged fermions (Dirac particles) are represented by four-component spinors.

### 1.3.1 Dirac Neutrinos

It is known that the lepton number is conserved, on the other hand, as a was indicated in previous paragraphs, we can make $M_N = 0$ in the Eq.(1.5) results equivalent to regard lepton number conservation. Applying it in Eq.(1.5) and considering that sterile neutrinos are three, so we can recognize them as the right-handed component of a neutrino four spinor field then the Dirac mass term can be diagonalized with two $3 \times 3$ unitary matrices, $U^\nu$ and $U_R^\nu$ as:

$$U_R^{\nu\dagger}\, M_D\, U^\nu = \hat{M}_D \tag{1.11}$$

[2]see the appendix E

The lagrangian density of the neutrino take the form:

$$-\mathcal{L}_{M_\nu} = \sum_{i=1}^{3} m_i \overline{\psi}_{D_i,\nu}\psi_{D_i,\nu}, \tag{1.12}$$

we can identify:

$$\psi_{D_i,\nu} = \left(U^{\nu\dagger}\overline{\psi}'_{L,\nu}\right)_i + \left(U_R^{\nu\dagger}\overline{\psi}'_{s,\nu}\right)_i, \tag{1.13}$$

remember that $\psi_{L,\nu}$ represents the active neutrino and $\psi_{s,\nu}$ represents the sterile one. Therefore, the weak-doublet components of the neutrino fields are:

$$\psi'_{L_i,\nu} = P_L \sum_{j=1}^{3} U^{\nu}_{ij}\psi_{D_j,\nu}, \qquad i = 1, 2, 3. \tag{1.14}$$

It is important to point out that both the matter content and the assumed symmetries are different. Moreover, there is no explanation to the fact that neutrino masses must be much lighter compared to charged fermion masses which acquire their mass using the same mechanism.

### 1.3.2 Brief Summary of the See-saw Mechanism

In this case the mass eigenvalues of $M_N$ is much higher than the ones of electroweak symmetry breaking $\langle\phi\rangle$. When we diagonalize the $M_\nu$ matrix, as a consequence, we obtain three light neutrinos, $\nu_\ell$, and $m$ heavy ones:

$$-\mathcal{L}_{M_\nu} = \frac{1}{2}\overline{\psi}_{\nu_\ell}M^\ell\psi_{\nu_\ell} + \frac{1}{2}\overline{\psi}_N M^h \psi_N, \tag{1.15}$$

where

$$M^\ell \simeq -U_\ell^T M_D^T M_N^{-1} M_D U_\ell, \qquad M^h \simeq U_h^T M_N U_h, \tag{1.16}$$

and

$$U^\nu \simeq \begin{bmatrix} \left(1 - \frac{1}{2}M_D^\dagger M_N^{*-1} M_N^{-1} M_D\right) U_\ell & M_D^\dagger M_N^{*-1} U_h \\ -M_N^{-1} M_D U_\ell & \left(1 - \frac{1}{2} M_N^{-1} M_D M_D^\dagger M_N^{*-1}\right) U_h \end{bmatrix} \tag{1.17}$$

$U_\ell$ and $U_h$ are unitary matrices respectively. Moreover, we can see from Eq. (1.16) that these matrices are proportional to $M_N$ and the lighter ones are proportional to $M_N^{-1}$. This is called the See-Saw Mechanism. In addition, from (1.17) the heavy states are right-handed while the light ones are in most cases left-handed. In this way the light and the heavy states are Majorana particles. The See-Saw Mechanism can be consered as a particular example of a whole theory which to low energy we obtain the ESM plus three light Majorana like neutrinos.

### 1.3.3 New Physics and the Neutrino Masses

From the above discussion, there are several reasons to think that the ESM isn't a complete picture of nature. If any of the extensions of the SM is indeed realized in nature, the SM must be thought of as an effective low energy theory. It means that it is a valid approximation up to the scale $\Gamma_{NP}$ (see below) which characterizes the new physics (NP: New Physics).

Regarding to the SM as an effective low enegy theory, without changing the gauge symmetry group, $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, the fermionic spectrum and the hierarchy of SSB (Spontaneous Symmetry Breaking) as valid ingredients to describe nature at energies $E \ll \Gamma_{NP}$. Nevertheles, the modifications of the SM predictions are given by small effects which are proportional to powers of $E/\Gamma_{NP}$. The difference between the ESM, as a whole description of nature and like a low energy effective theory is which in the latter one, it must regard no-renormalizable terms.

In fact, there is a single set of dimension-five operators terms that is made of SM fields which is consistent with the gauge symmetry. This term is given by the Weinberg operator[13]:

$$\mathcal{O}_5 = \frac{X^{\nu}_{ij}}{\Gamma_{NP}} \left(\overline{\Psi}_{Li}\tilde{\phi}\right)\left(\tilde{\phi}^T \Psi^c_{Lj}\right) + H.C. \tag{1.18}$$

$X^{\nu}_{ij}$ is a constant elements matrix, furthermore, the equation 1.18 violates total lepton number by two units leading, after SSB, we arrive to:

$$\mathcal{L}_{M_\nu} = \frac{Y^{\nu}_{ij}}{2}\frac{v^2}{\Gamma_{NP}}\overline{\psi}_{Li,\nu}\psi^c_{Lj,\nu} + H.C., \tag{1.19}$$

where the neutrinos masses are given by:

$$(\mathcal{M}_\nu)_{ij} = \frac{X^{\nu}_{ij}}{2}\frac{v^2}{\Gamma_{NP}}. \tag{1.20}$$

Since Eq. (1.20) would arise in a generic extension of the SM, we learn that neutrino masses are very likely to appear if there is new physics. Moreover, if neutrino masses arise effectively from nonrenomalizable terms, we gain an understanding not only for the existence of neutrino masses but also for their smallness. The scale of neutrino masses is suppressed by $v/\Gamma_{NP}$, compared to the scales of charged fermion masses.

The equation (1.19) break not only total lepton number but also the lepton flavor symmetry, hence we should expect lepton mixing and CP violation[3].

## 1.4 Testing of the ESM

In addition to cross-sections, asymmetries, parity violation, W and Z decays, there is a large number of experiments and observables testing the flavor structure of the SM.

We present the results of global fits to experimental data. The values for $m_t$ (quark top mass), $\nu$-lepton scattering, the weak charge ( In the SM: $2T_3 - 4\,Q\,\sin^2\theta_W$) of the electron, the proton, cesium, thallium, the weak mixing angle extracted from ATLAS, CMS, D0 experiments; the muon anomalous magnetic moment, and the $\tau$ lifetime are listed in the table 1.1. $M_H$ (Higgs boson mass) is our average of the LHC combination from Run 1, $M_H = 125.09 \pm 0.21$ *stat.* $\pm 0.11$ *syst.* *GeV*, with $M_H = 124.98 \;\pm 0.19$ *stat.* $\pm 0.21$ *syst.* *GeV* from ATLAS and $M_H = 125.26 \;\pm 0.29$ *stat.* $\pm 0.08$ *syst.* *GeV* from CMS at Run 2, where conservatively is treated the smallest systematic error as common among the three determinations.

[3]M. C. Gonzalez - Garcia, Neutrino masses and mixing evidence and implication

| Quantity | Experimental value | SM (predictions) |
|---|---|---|
| $m_t$ [GeV] | $172.89 \pm 0.59$ | $173.19 \pm 0.55$ |
| $M_W$ [GeV] | $80.387 \pm 0.016$ | $80.361 \pm 0.006$ |
| | $80.376 \pm 0.033$ | |
| | $80.370 \pm 0.019$ | |
| $\Gamma_W$ [GeV] | $2.046 \pm 0.049$ | $2.090 \pm 0.001$ |
| | $2.195 \pm 0.083$ | |
| $M_Z$ [GeV] | $91.1876 \pm 0.0021$ | $91.1882 \pm 0.0020$ |
| $\Gamma_Z$ [GeV] | $2.4955 \pm 0.0023$ | $2.4942 \pm 0.0009$ |
| $\Gamma_{e^+e^-}$ [MeV] | $83.942 \pm 0.085$ | $83.964 \pm 0.009$ |
| $\Gamma_{\mu^+\mu^-}$ [MeV] | $83.941 \pm 0.085$ | $83.963 \pm 0.009$ |
| $\Gamma_{\tau^+\tau^-}$ [MeV] | $83.759 \pm 0.085$ | $83.780 \pm 0.009$ |
| $M_H$ [GeV] | $125.14 \pm 0.15$ | $125.14 \pm 0.15$ |
| $g_V^{\nu_e}$ | $-0.040 \pm 0.015$ | $-0.0398 \pm 0.0001$ |
| $g_A^{\nu_e}$ | $-0.507 \pm 0.014$ | $-0.5063$ |
| $a_\mu$ | $(1165920.91 \pm 0.63) \times 10^{-9}$ | $(1165918.36 \pm 0.44) \times 10^{-9}$ |
| $\hat{s}_Z^2$ | $0.2299 \pm 0.0043$ | $0.23122 \pm 0.00003$ |
| $g_{AV}^{eu} + 2g_{AV}^{ed}$ | $0.4914 \pm 0.0031$ | $0.4950$ |
| $2g_{AV}^{eu} - g_{AV}^{ed}$ | $-0.7148 \pm 0.0068$ | $-0.7194$ |
| $g_{AV}^{ee}$ | $0.0190 \pm 0.0027$ | $0.0226$ |
| $Q_W(e)$ | $-0.0403 \pm 0.0053$ | $-0.0476 \pm 0.0002$ |
| $Q_W(p)$ | $0.0719 \pm 0.0045$ | $0.0711 \pm 0.0002$ |
| $Q_W(Cs)$ | $-72.62 \pm 0.43$ | $-73.23 \pm 0.01$ |
| $Q_W(Tl)$ | $-116.4 \pm 3.6$ | $-116.87 \pm 0.02$ |
| $\tau_\tau[fs]$ | $290.75 \pm 0.36$ | $290.39 \pm 2.17$ |

Table 1.1: Experimental datas and the SM predictions.

According to the values placed in this table we can also mention the following:

1. The first $M_W$ and $\Gamma_W$ values were obtained from the Tevatron, the second ones from LEP 2, while the third $M_W$ is from ATLAS.
2. The $\tau_\tau$ value is the $\tau$ lifetime world average computed by combining the direct measurements with values derived from the leptonic branching ratios.
3. The $M_Z$, $\Gamma_Z$, $\hat{s}_Z^2$ and $\Gamma_{\ell^+\ell^-}$ have been performed at LEP 1 and SLAC.
4. The muon anomalous magnetic momento is dominated by the final result of the BNL E821 collaboration[4]
5. The world averages for $g_A^{\nu_e}$ and $g_V^{\nu_e}$ are dominated by the CHARM II[5].
6. $g_{AV}^{eu} + 2g_{AV}^{ed}$, $g_{AV}^{ee}$ and $2g_{AV}^{eu} - g_{AV}^{ed}$ were obtained from the CMS and ATLAS collaboration.

## 1.5 Independent parameters of the SM

The SM Lagrangian density contains many parameters. However, not all these parameters can have physical meaning. One is always free to re-define the SM fields in an arbitrary manner. By

[4]G. Bennett et al. (Muon g-2), Phys. Rev. Lett. 92, 161802 (2004)
[5]P. Vilain et al. (CHARM-II), Phys. Lett. B 335, 246 (1994).

suitable re-definitions, one can remove some of the apparent parameter freedom and identify the true physical independent parametric degrees of freedom.

The model that unifies three of the fundamental forces and describes all matter in the form of quarks and leptons, has about 18 free parameters which are not predicted by the theory.

The following table 1.2 shows the 18 independent parameters of the SM. As can be seen, most of the model's free parameters belong to the quark sector[6]:

| Quantity | Simbol | Valou |
|---|---|---|
| Gauge coupling constants | $\alpha$ | $7.297352\ 5664(17) \times 10^{-3}$ |
| | $\sin^2 \theta_W$ | $0.23122(4)$ |
| | $\alpha_s$ | $0.1181(11)$ |
| Gauge bosons masses | $m_W$ | $80.379 \pm 0.012$ GeV |
| | $m_H$ | $125.10 \pm 0.14$ GeV |
| Fermion masses | $m_e$ | $0.5109989461 \pm 0.0000000031$ MeV |
| | $m_\mu$ | $105.6583745 \pm 0.0000024$ MeV |
| | $m_\tau$ | $1776.86 \pm 0.12$ MeV |
| | $m_\mu$ | $2.16^{+0.49}_{-0.26}$ MeV |
| | $m_c$ | $1.27 \pm 0.02$ GeV |
| | $m_t$ | $172.9 \pm 0.4$ GeV |
| | $m_d$ | $4.67^{+0.48}_{-0.17}$ MeV |
| | $m_s$ | $93^{+11}_{-5}$ MeV |
| | $m_b$ | $4.18^{+0.03}_{-0.02}$ GeV |
| CKM Matriz parameters | $J_{CKM}$ | $(3.18 \pm 0.15) \times 10^{-5}$ |
| | $\|V_{\mu d}\|$ | $0.97420 \pm 0.0021$ |
| | $\|V_{\mu s}\|$ | $0.2243 \pm 0.0005$ |
| | $\|V_{\mu b}\|$ | $(3.94 \pm 0.36) \times 10^{-3}$ |

Table 1.2: The SM parameters.

[6]Particle Data Group 2020, (http://pdg.lbl.gov)

## 1.6 Questions that can't be answered by the SM

The ESM is one of the most successful theories in physics. So far, it withstands all tests and has been experimentally verified with great accuracy. One of its latest triumphs is the discovery of a neutral scalar particle which appears to have the properties predicted by the SM, the Higgs Boson. However, there are several phenomenas that cannot be explained within the energy scale of the SM and hence require the existence of some kind of yet undiscovered physics, commonly referred to as new physics or physics beyond the SM (BSM).

Within the SM, neutrinos are treated as massless, but observation of neutrino oscillation demands that neutrinos in fact do have a non-vanishing mass, although a very small one.

Another challenge for the SM is the so-called hierarchy problem: The SM gives no explanation for the enormous difference between the electroweak scale (until about[1] 200 GeV), the scale at which electroweak and strong forces become equally strong (due to the running coupling constants) which is of the order of $10^{16}$ GeV and the Planck scale of $\sim 10^{19}$ GeV, at which also the gravitational interaction becomes as strong as the other forces.While the masses of the fundamental particles can be generated via the Higgs-mechanism in electroweak symmetry breaking, the theory gives no explanation for the large range of the masses. Moreover, additional particles are needed in order to cancel diverging loop-corrections to the Higgs mass.

There is also no explanation why, within the SM, there are three generations of fundamental fermions.

The origin of the matter-antimatter asymmetry in the universe is another open question in particle physics: If at the big bang, particles and antiparticles were created in the same amount, they should all have annihilated each other, however, the annihilation appears to be asymmetric as there is today only matter observed in the universe while the antimatter has disappeared. This requires CP violation by an amount that cannot be accommodated in the SM.

Finally, cosmological and astrophysical observations lead to the conclusion, that radiation and matter made of SM particles only account for about 5 % of the mass and energy content in the universe. Roughly 27 % are attributed to non-luminous dark matter and the remaining roughly 68 % are so-called dark energy. Neither of these last two components finds any explanation within the SM.

In the next chapter we will study one of the so-called left-right symmetric models, which are extensions of the standard model, with the aim of trying to explain some of the many questions that can not be justified with the SM. The left-right symmetric model is built by modifying the electroweak gauge group. There may be added a right-handed $SU(2)_R$ group and the charge on $U(1)$ is modified to a new charge denoted by $\tilde{Y} = B - L$, where $B$ : Barionic number and $L$ : Leptonic number, which will be explained later.

$$G_{LR} = SU(2)_L \otimes SU(2)_R \otimes U(1)_{\tilde{Y}}$$

This kind of model was first suggested by physicists Jogesh Pati and Abdus Salam[4], in an attempt to introduce left-right symmetry.

# Chapter 2

# The Left - Right Model with gauge symmetry group $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L} \otimes \mathcal{P}$

## 2.1 Introduction

It is known that the SM violates the parity which can understand it as left-right asymmetry of elementary particles. One way to study and understand the left-right asymmetry is to enlarge the gauge symmetry group of the ESM into a left-right (LR) symmetry one and then, by applying spontaneously breaking mechanism, to recover the ESM symmetry structure. For instance, in left-right symmetric models [4], the gauge $SU(2)_R$ group is introduced to maintain parity invariance at high energy scales with new scalar multiples: Two doublets and two bi-doublets scalar fields. The symmetry group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ of LR symmetric models can be subgroup of grand unified symmetry groups such as $SO(10)$[14], $E_6$[15] or superstring models[16]. In the framework of LR symmetric models, the left-handed fermions are placed in the $SU(2)_L$ doublets like they are in the ESM while the right-handed fermions (where appearing the right-handed neutrinos which are part of the leptons) are placed in the $SU(2)_R$ isospin doublets. Subsequently, following the usual process, the LR symmetry is spontaneously broken down to the electroweak symmetry of the SM using the known Higgs Mechanism. According to the bibliography, exist several variants of LR symmetric models which have been proposed such as these references: [6]-[17].

The idea of the non-conservation of parity in the SM was proposed in a classic paper[18] by Lee and Yang where they proposed the existence of additional fermions of opposite chirality to the SM ones to make the world left-right symmetric at high energies. Moreover, the smallness of neutrino mass can be explained via a see-saw mechanism, where the neutrino is a Majorana fermion like by regarding in the scalar sector triplets one. These models can also be useful by explaining the Dark Matter problem[19], neutrino oscillations, as well as different neutrino physics anomalies such as solar neutrino deficit and atmospheric neutrino anomaly[20]. In summary, the existence of right particles appear naturally in models which go beyond the SM, like GUT and string theories[21]. The right particles masses, which are unknown experimentally, can be at or below the TeV scale. The agreement of the LR models with electroweak precision data have been studied in [22].

In this chapter I start giving an overview of our left-right symmetric model with gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes \mathcal{P}$, where the parity operator $\mathcal{P}$ assures me the left-right symmetry of the model; therefore parity is preserved from the start. We have also regarded two extra left right models in order to compare it with our model (These articles are of Goran Senjanovic's [7] and Mohapatra's [4]). In addition, we will give some details about the most important sectors in our model, by defining the theoretical framework through the gauge group considered in this research work and showing how the particles (fermions) are placed into multiplets according to the gauge symmetry group. Within the most important sectors we are going to show the gauge sector through the covariant derivaties of the scalar sector (kinetic part), however, the gauge bosons' physical states and masses are calculated in the next chapter by applying the Higgs mechanics and the SSB[10, 8]. Moreover, we analize the most general potential (which respects several symmetry) and the Yukawa sector that is built respecting the symmetries of the model.

Remember that LRSM has been studied extensively since the 1970s; as one example of an early review see [4, 5]. The most common and popular version of the model is called the minimal LRSM and is defined by its scalar sector, where the usual SM Higgs doublet is replaced by a bi-doublet and two complex triplets which are introduced into the theory. One of the complex triplets transforms under $SU(2)_R$ and the other one under $SU(2)_L$. In addition, the VEV of the right-handed triplet is large, resulting in the gauge bosons $Z'$ and $W_R^{\pm}$ whose masses are larger than the SM gauge bosons masses.

To conclude, I am exploring an alternative symmetry breaking pattern (six VEV's), where the vevs $v_R$ is considered to be the largest one. This version of the model includes two doublets and two bi-doublets, within the scalar sector, in order to give large masses to $Z'$ and $W'^{\pm}$ bosons, which are the new vectorial bosons proposed by the model, and also, give masses to the other particles, this includes the light leptons and the quarks. Many features of the model are as similar as the minimal LRSM, however, the scalar sector that I have explored has not been worked before.

## 2.2 Particles Classification

### 2.2.1 Leptonic Sector

The left and right leptons (charged and neutral) are represented by doublets, using the fundamental representation of the symmetry group $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L} \otimes \mathcal{P}$[23]:

$$L_l \equiv \tfrac{1}{2}(1-\gamma_5)\begin{pmatrix}\psi_{\nu_l}\\ \psi_l\end{pmatrix} = \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L ; \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L ; \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L \sim (\mathbf{2}_L, \mathbf{1}_R, -1)$$

$$R_l \equiv \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\psi_{\nu_l}\\ \psi_l\end{pmatrix} = \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_R ; \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_R ; \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_R \sim (\mathbf{1}_L, \mathbf{2}_R, -1) \tag{2.1}$$

where the quantum numbers in parentheses represent the third component of the left weak isospin (right), $T_{3L}(T_{3R})$, and the hypercharge $B-L$, which is introduced to implement quark and lepton correspondence, since they are distinguished only by the $B-L$ quantum number[24][25]. These quantum numbers are related to the electric charge by the relation of Gell Mann-Nishijima of the model:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} \tag{2.2}$$

The quantum numbers corresponding to the representation (2.1) are summarized in the following table:

| Leptón | $T_{3L}$ | $T_{3R}$ | $\frac{B-L}{2}$ | $Q$ |
|---|---|---|---|---|
| $\nu_{eL}$, $\nu_{\mu L}$, $\nu_{\tau L}$ | 1/2 | 0 | -1/2 | 0 |
| $e_L$, $\mu_L$, $\tau_L$ | -1/2 | 0 | -1/2 | -1 |
| $\nu_{eR}$, $\nu_{\mu R}$, $\nu_{\tau R}$ | 0 | 1/2 | -1/2 | 0 |
| $e_R$, $\mu_R$, $\tau_R$ | 0 | -1/2 | -1/2 | -1 |

**Table 2.1 :** *Quantum numbers for leptons at the left - right model.*

Because the fields are transformed according to the local symmetry group $SU(2)_R \times SU(2)_L \times U(1)_{B-L} \otimes \mathcal{P}$. These fields transform non trivially under different $SU(2)$ transformation, in addition, the leptonic lagrangean density must remain invariant under this transformation, and as a consequence of the gauge invariant, new bosonic fields are introduced:

$W^R_{\mu k}$, $W^L_{\mu k}$ with $k = 1, 2, 3$ associated with groups $SU(2)_R$ y $SU(2)_L$ respectively, and $B_\mu$ related to the group $U(1)_{B-L}$, the ordinary derivative is replaced by the covariant derivative:

$$\begin{aligned} D^L_\mu &\equiv \partial_\mu + \frac{ig_L}{2}\,\bar{\tau}.\bar{W}^L_\mu - \frac{ig'}{2}\,B_\mu \\ D^R_\mu &\equiv \partial_\mu + \frac{ig_R}{2}\,\bar{\tau}.\bar{W}^R_\mu - \frac{ig'}{2}\,B_\mu \end{aligned} \tag{2.3}$$

however, making use of the parity invariance, $\mathcal{P}$, the covariant derivaties should be written as:

$$\begin{aligned} D^L_\mu &\equiv \partial_\mu + \frac{ig}{2}\,\bar{\tau}.\bar{W}^L_\mu - \frac{ig'}{2}\,B_\mu \\ D^R_\mu &\equiv \partial_\mu + \frac{ig}{2}\,\bar{\tau}.\bar{W}^R_\mu - \frac{ig'}{2}\,B_\mu \end{aligned} \tag{2.4}$$

where: $g_R = g_L = g$ y $g_{B-L} \equiv g'$ are the coupling constants of the weak and hypercharge symmetry. To observe that we need making $g_R = g_L$, with the aim that the theory must be invariant by parity $\mathcal{P}$, where fields transform by Parity as follows[26]:

$$L_l \overset{\mathcal{P}}{\leftrightarrow} R_l \qquad ; \qquad \bar{W}_L \overset{\mathcal{P}}{\leftrightarrow} \bar{W}_R$$

that is, we impose a generalized parity under which

$$g_L \leftrightarrow g_R,\ W_{L\mu} \leftrightarrow W^\mu_R,\ f_L \leftrightarrow f_R,\ \chi_L \leftrightarrow \chi_R, \Phi_i \leftrightarrow \Phi^\dagger_i, \tilde{\Phi}_i \leftrightarrow \tilde{\Phi}^\dagger_i, \tag{2.5}$$

where $\tilde{\Phi}_i = \tau_2 \Phi^*_i \tau_2$; $W_{\mu L,R}$ are the vectorial gauge bosons of the $SU(2)_{L,R}$ gauge symmetry, respectively, $f$ denotes a quark or a lepton doublet, and $\Phi_i$ and $\chi_{L,R}$ are the scalar multiplets that will be defined in the scalar sector of the model, see later sections.

As we mentioned in the previous paragraphs, the invariance under $\mathcal{P}$ implies equality of gauge couplings $g_L = g_R \equiv g$ at the energy at which these symmetries are realized. Under this condition, the model has only two gauge couplings, $g$ and $g'$; however, as a result of running couplings, we will have $g_L \neq g_R$[27], we consider in this work the case when these two couplings are equal at any energy scale, but this has to be seen just as an approximation.

In the next chapter (section 3.3.1), we are justifying the relations between the coupling constants which are given by the following:

$$g \;=\; \frac{e}{sin\theta}, \qquad g' \;=\; \frac{e}{\sqrt{\cos 2\theta}} \tag{2.6}$$

We may define the following parameter:

$$r_\theta^2 \equiv \frac{g'^2}{g^2} = \frac{s_\theta^2}{c_{2\theta}^2} = \frac{s_\theta^2}{1-2s_\theta^2}, \tag{2.7}$$

this last expression implies that $s_\theta^2 < 1/2$, to this expression we can call matching condition. The weak mixing angle $\theta$ is not equal to the weak mixing angle of the SM, $\theta_W$, not even at any energy scale. However, at a certain energy scale, which satisfy the matching condition the prediction of the LR model must coincide with those of the SM.

In others words, this implies only that the energy scale at which $g_L(\mu) = g_R(\mu)$ must be below the scale at which:

$$s_\theta^2(\Lambda) \;=\; 1/2, \qquad \mu < \Lambda$$

Notice that the equation (2.6) can be written like:

$$\frac{1}{e^2} \;=\; \frac{2}{g^2} + \frac{1}{g'^2}, \tag{2.8}$$

which implies a matching condition with the SM that is valid at a given energy:

$$\frac{1}{g_Y^2} \;=\; \frac{1}{g^2} + \frac{1}{g'^2}, \tag{2.9}$$

where $g_Y$ is the coupling constant relates to the gauge group $U(1)_Y$, similar to the SM. Remember that in the SM, it is known:

$$g_Y/g = \tan\theta_W \tag{2.10}$$

Returning to the Leptonic sector, the leptonic lagrangian density, invariant under the symmetry $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L} \otimes \mathcal{P}$ can be built by the covariant derivatives(2.3):

$$\mathcal{L}^{lep}(x) = i\,\left\{\bar{L}_l(x)\gamma^\mu D_\mu^L L_l(x) + \bar{R}_l(x)\gamma^\mu D_\mu^R R_l(x)\right\} + h.c. \tag{2.11}$$

We can make $L \leftrightarrow R$, then the lagrangian density turns out to be invariant pointing out a parity symmetry appearing.

### 2.2.2 Scalar Sector Multiplets

What differentiates our model from other similar let-right models is the scalar sector. This sector consists of two bi-doublets transforming as $(\mathbf{2}, \mathbf{2}^*, 0)$:

$$\Phi_1 = \begin{pmatrix} \phi_1^0 & \eta_1^+ \\ \phi_1^- & \eta_1^0 \end{pmatrix}, \; \Phi_2 = \begin{pmatrix} \phi_2^0 & \eta_2^+ \\ \phi_2^- & \eta_2^0 \end{pmatrix}, \tag{2.12}$$

where the first bi-doublet, $\Phi_1$[1], couple with leptons and the other one, $\Phi_2$, couple with quarks. These bi-doublets will generate masses to the leptons (including neutrinos) and quarks respectivily. In addition, the scalar sector also consists of two doublets $\chi_L \sim (\mathbf{2_L}, \mathbf{1_R}, +1)$ and $\chi_R \sim (\mathbf{1_L}, \mathbf{2_R}, +1)$ to break the parity and the gauge symmetry down to $U(1)_Q$, as in [28, 7].

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}, \tag{2.13}$$

We are going to suposse that the vacuum expectation values, VEV, exist, and they are given by:

$$< \Phi_1 >= \tfrac{1}{\sqrt{2}} \begin{pmatrix} k_1 & 0 \\ 0 & k_1' \end{pmatrix}, \quad < \Phi_2 >= \tfrac{1}{\sqrt{2}} \begin{pmatrix} k_2 & 0 \\ 0 & k_2' \end{pmatrix}, \tag{2.14}$$

and

$$\langle \chi_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle \chi_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \tag{2.15}$$

The neutral components of the scalars can be written like the following expression:

$$\langle x_i \rangle = \frac{1}{\sqrt{2}} (v_i + R_i + iI_i) e^{i\theta_i} \tag{2.16}$$

$v_i, \theta_i$: real numbers; $R_i, I_i$ Hermitian fields, but, here we are going to consider all VEVs real, that is, $\theta_i = 0$ for all $i$ running over scalar multiplets.

Considering complex VEVs implies spontaneous *CP* violation [29]. Moreover, remember that are the neutral scalar fields which gain non-zero vev, because the vacuum state must be neutral, the electric charge must be conserved, while the corresponding charged scalars fields do not.

## 2.3 The Scalar Potential

First, we are going to regard the most general scalar potential, invariant under the gauge symmetry group:

$$V = V^{(2)} + V^{(4a)} + V^{(4b)} + V^{(4c)} + V^{(4d)} + V^{(4e)} \tag{2.17}$$

[1]In appendix H, the assignment of electric charge to the scalar fields of each multiplet is detailed.

Where:

$$V^{(2)} = \frac{1}{2}\sum_{i,j}^{2}\Big[\mu_{ij}^2\,Tr(\Phi_i^\dagger\,\Phi_j) + \tilde{\mu}_{ij}^2\,Tr(\tilde{\Phi}_i^\dagger\,\Phi_j) + H.C.\Big] + \mu_{LR}^2(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R), \quad (2.18)$$

$$V^{(4a)} = \frac{1}{2}\sum_{i,j}^{2}\Big[\lambda_{ij}\,Tr(\Phi_i^\dagger\,\Phi_j)^2 + \tilde{\lambda}_{ij}\,Tr(\tilde{\Phi}_i^\dagger\,\Phi_j)^2 + H.C.\Big], \quad (2.19)$$

$$V^{(4b)} = \frac{1}{2}\sum_{i,j}^{2}\Big[\lambda'_{ij}\,(Tr\Phi_i^\dagger\,\Phi_j)^2 + \tilde{\lambda}'_{ij}\,(Tr\tilde{\Phi}_i^\dagger\,\Phi_j)^2 + H.C.\Big], \quad (2.20)$$

$$V^{(4c)} = \rho_{12}\,Tr(\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2) + \tilde{\rho}_{12}\,Tr(\tilde{\Phi}_1^\dagger\Phi_1\tilde{\Phi}_2^\dagger\Phi_2), \quad (2.21)$$

$$\begin{aligned} V^{(4d)} &= \frac{1}{2}\Bigg[\sum_{i,j}^{2}(\Lambda_{ij}\,Tr\Phi_i^\dagger\Phi_j + \tilde{\Lambda}_{ij}\,Tr\tilde{\Phi}_i^\dagger\Phi_j)(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R) \\ &+ \bar{\Lambda}_{ij}(\chi_L^\dagger\Phi_i\Phi_j^\dagger\chi_L + \chi_R^\dagger\Phi_i^\dagger\Phi_j\chi_R) + \Omega_{ij}(\chi_L^\dagger\tilde{\Phi}_i\Phi_j^\dagger\chi_L + \chi_R^\dagger\tilde{\Phi}_i^\dagger\Phi_j\chi_R) \\ &+ \bar{\Lambda}'_{ij}(\chi_L^\dagger\tilde{\Phi}_i\tilde{\Phi}_j^\dagger\chi_L + \chi_R^\dagger\tilde{\Phi}_i^\dagger\tilde{\Phi}_j\chi_R) + \Omega'_{ij}(\chi_L^\dagger\Phi_i\tilde{\Phi}_j^\dagger\chi_L + \chi_R^\dagger\Phi_i^\dagger\tilde{\Phi}_j\chi_R) + H.C.\Bigg] \end{aligned} \quad (2.22)$$

$$V^{(4e)} = \lambda_{LR}\Big[(\chi_L^\dagger\chi_L)^2 + (\chi_R^\dagger\chi_R)^2\Big]. \quad (2.23)$$

Where we have omitted the redundant terms, for instance:

$$\text{Tr}\left(\tilde{\Phi}_i^\dagger\tilde{\Phi}_j^\dagger\right) = Tr(\Phi_i^\dagger\,\Phi_j), \qquad i,j = 1,2. \quad (2.24)$$

Because when they acquire VEVs, they reproduce the same results, that is:

$$\begin{aligned} \text{Tr}\left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_1^\dagger\right) &= Tr(\Phi_1^\dagger\,\Phi_1) = \frac{k_1'^2}{2} + \frac{k_1^2}{2}, \\ \text{Tr}\left(\tilde{\Phi}_2^\dagger\tilde{\Phi}_2^\dagger\right) &= Tr(\Phi_2^\dagger\,\Phi_2) = \frac{k_2'^2}{2} + \frac{k_2^2}{2}, \\ \text{Tr}\left(\tilde{\Phi}_1^\dagger\tilde{\Phi}_2^\dagger\right) &= Tr(\Phi_1^\dagger\,\Phi_2) = \frac{k_1'k_2'}{2} + \frac{k_1k_2}{2}, \\ \text{Tr}\left(\tilde{\Phi}_2^\dagger\tilde{\Phi}_1^\dagger\right) &= Tr(\Phi_2^\dagger\,\Phi_1) = \frac{k_1'k_2'}{2} + \frac{k_1k_2}{2}, \end{aligned} \quad (2.25)$$

Remember that $\tilde{\Phi}_i = \tau_2\Phi_i^*\tau_2$. This potential is discussed in the appendix F, where we have found its minimal value and the constraint equations. Although we have not carried out any calculations with this potential, The information that can be obtained through their constrain equations will serve to develop some later phenomenological calculations, through other research works.

### 2.3.1 Transformation rules of the Bi-doublets Scalar Fields

A bidoublet $\Phi$ transforms under the $SU(2)_L \otimes SU(2)_R$[2] symmetry as:

$$\begin{aligned} \Phi &\to U_L\Phi\,U_R^\dagger, \\ \Phi^\dagger &\to U_R\Phi^\dagger\,U_L^\dagger, \\ \tilde{\Phi} &\to U_L\tilde{\Phi}\,U_R^\dagger, \\ \tilde{\Phi}^\dagger &\to U_R\tilde{\Phi}^\dagger\,U_L^\dagger, \end{aligned} \quad (2.26)$$

[2] In principle, it must be transformed according to gauge group $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$, however, the hypercharge value of $\Phi$, respect to the gauge group $U(1)_{B-L}$ is zero, that is $B - L = 0$

for instance, we can use the Yukawa sector to demostrate how the bi-doublets are transformed:

$$-\mathcal{L}_Y = \overline{L'}(G\Phi_1 + F\tilde{\Phi}_1)R' + h.c. \tag{2.27}$$

1. For the case of the bi-doublet $\Phi_1$:

$$-\mathcal{L}_Y = \overline{L'}\, G\Phi'_1\, R' + \cdots , \tag{2.28}$$

however, the leptonic doublet:

$$\begin{array}{rcl} L' &=& e^{i\,\bar{\tau}\cdot\bar{\alpha}_L} L = U_L\, L \quad \rightarrow \quad \overline{L'} = \overline{L}\, U_L^\dagger \\ R' &=& e^{i\,\bar{\tau}\cdot\bar{\alpha}_R}\, R = U_R\, R \quad \rightarrow \quad \overline{R'} = \overline{R}\, U_R^\dagger \end{array}$$

replacing on the equation (2.28), we obtain:

$$\begin{aligned} -\mathcal{L}_Y &= \overline{L}\, U_L^\dagger\, G\, \Phi'_1\, U_R R + \cdots \\ &= \overline{L}\, G\, \underbrace{U_L^\dagger\, \Phi'_1\, U_R}_{=\Phi_1} R + \cdots \\ &= \overline{L}\, G\, \Phi_1 R + \cdots \end{aligned} \tag{2.29}$$

we can observe:

$$\Phi_1 = U_L^\dagger\, \Phi'_1\, U_R \quad \rightarrow \quad \Phi'_1 = U_L \Phi_1 U_R^\dagger \tag{2.30}$$

where we are use the fact that:

$$U_L^\dagger U_L = U_R^\dagger U_R = I, \quad \text{unitary operator} \tag{2.31}$$

Also, from the equation (2.30) we get the transformation rule of $\Phi_1^\dagger$:

$$\Phi_1'^\dagger = U_R \Phi_1^\dagger U_L^\dagger \tag{2.32}$$

2. For the case of the bi-doublet $\tilde{\Phi}_1$:

$$-\mathcal{L}_Y = \overline{L'}\, F\tilde{\Phi}'_1\, R' + \cdots , \tag{2.33}$$

in the same way as the previous item, we have:

$$\begin{aligned} -\mathcal{L}_Y &= \overline{L}\, U_L^\dagger\, F\, \tilde{\Phi}'_1\, U_R R + \cdots \\ &= \overline{L}\, F\, \underbrace{U_L^\dagger\, \tilde{\Phi}'_1\, U_R}_{=\tilde{\Phi}_1} R + \cdots \\ &= \overline{L}\, F\, \tilde{\Phi}_1 R + \cdots \end{aligned} \tag{2.34}$$

we can observe:

$$\tilde{\Phi}_1 = U_L^\dagger\, \tilde{\Phi}'_1\, U_R \quad \rightarrow \quad \tilde{\Phi}'_1 = U_L \tilde{\Phi}_1 U_R^\dagger \tag{2.35}$$

then

$$\tilde{\Phi}_1'^\dagger = U_R \tilde{\Phi}_1 U_L^\dagger \tag{2.36}$$

both bi-doublets, $\Phi_1$ and $\tilde{\Phi}_1$ transform of the same way. The same occurs with the bi-doublet $\Phi_2$ and $\tilde{\Phi}_2$, which are coupled with quarks.

**Another way to demonstrate:** $\tilde{\Phi}'_1 \;=\; U_L \tilde{\Phi}_1 U_R^\dagger$

From the previous results:

$$\Phi' \;=\; U_L \Phi U_R^\dagger$$

taking the complex conjugate:

$$\Phi'^* \;=\; U_L^* \Phi^* U_R^T$$

multiplying in both sides by $\tau_2$, we have:

$$\underbrace{\tau_2\, \Phi'^*\, \tau_2}_{\tilde{\Phi}'} \quad = \quad \tau_2\, U_L^*\, \tau_2\; \underbrace{\tau_2\, \Phi^*\, \tau_2}_{=\tilde{\Phi}}\; \tau_2\, U_R^T\, \tau_2,$$

then:

$$\tilde{\Phi}' \quad = \quad \tau_2\, U_L^*\, \tau_2\, \tilde{\Phi}\, \tau_2\, U_R^T\, \tau_2, \tag{2.37}$$

we can define the unitary operators as follows:

$$U_R = e^{-i\bar{\beta}\cdot\bar{\tau}}, \qquad U_L = e^{-i\bar{\alpha}\cdot\bar{\tau}},$$

where: $\bar{\tau} = (\tau_1,\, \tau_2,\, \tau_3)$, $\tau_i$ : Pauli matrices. $\bar{\beta}$ and $\bar{\alpha}$ are real vectorials parameters. According to the Pauli matrices properties, we have:

$$\begin{aligned} \tau_2\, \tau_1^T\, \tau_2 &= -\tau_1, \\ \tau_2\, \tau_2^T\, \tau_2 &= -\tau_2, \\ \tau_2\, \tau_3^T\, \tau_2 &= -\tau_3, \end{aligned} \tag{2.38}$$

$$\begin{aligned} \tau_2\, \tau_1^*\, \tau_2 &= -\tau_1, \\ \tau_2\, \tau_2^*\, \tau_2 &= -\tau_2, \\ \tau_2\, \tau_3^*\, \tau_2 &= -\tau_3, \end{aligned} \tag{2.39}$$

then,

$$\begin{aligned} U_R = e^{-i\bar{\beta}\cdot\bar{\tau}} = I - i\bar{\beta}\cdot\bar{\tau} \quad \rightarrow \quad & U_R^T = I - i\bar{\beta}\cdot\bar{\tau}^T \\ \tau_2 U_R^T \tau_2 \quad &= \quad I - i\bar{\beta}\cdot \underbrace{\tau_2 \bar{\tau}^T \tau_2}_{=-\bar{\tau}} = I + i\bar{\beta}\cdot\bar{\tau} \\ \tau_2 U_R^T \tau_2 \quad &= \quad I + i\bar{\beta}\cdot\bar{\tau} = e^{i\bar{\beta}\cdot\bar{\tau}} = U_R^\dagger, \end{aligned} \tag{2.40}$$

in the same way:

$$\begin{aligned} U_L = e^{-i\bar{\alpha}\cdot\bar{\tau}} = I - i\bar{\alpha}\cdot\bar{\tau} \quad \rightarrow \quad & U_L^* = I + i\bar{\alpha}\cdot\bar{\tau}^* \\ \tau_2 U_L^* \tau_2 \quad &= \quad I + i\bar{\alpha}\cdot \underbrace{\tau_2 \bar{\tau}^* \tau_2}_{=-\bar{\tau}} = I - i\bar{\alpha}\cdot\bar{\tau} \\ \tau_2 U_L^* \tau_2 \quad &= \quad I - i\bar{\alpha}\cdot\bar{\tau} = e^{-i\bar{\alpha}\cdot\bar{\tau}} = U_L, \end{aligned} \tag{2.41}$$

finally, in the equation (2.42), we obtain:

$$\tilde{\Phi}' \quad = \quad U_L\, \tilde{\Phi}\, U_R^\dagger, \tag{2.42}$$

### 2.3.2 Analysis of the invariance of the most general scalar potential according to the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

According to the scalar potential given by equation (2.17) and from what is known from the previous section about the way in which bi-doublets are transformed according to $U(2)_L \otimes SU(2)_R$, we can analyze the invariance of the potential under this symmetry:

- From the expression:

$$V^{(2)} = \frac{1}{2}\sum_{i,j}^{2}\left[\mu_{ij}^2\, Tr(\Phi_i^\dagger\, \Phi_j) + \tilde{\mu}_{ij}^2\, Tr(\tilde{\Phi}_i^\dagger\, \Phi_j) + H.C.\right] + \mu_{LR}^2(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R)$$

  Analyzing the gauge invariance of this potential term:

$$\begin{aligned}
\mathrm{Tr}(\Phi'^\dagger\, \Phi') &= \mathrm{Tr}(U_R \Phi^\dagger \underbrace{U_L^\dagger\, U_L}_{=I} \Phi U_R^\dagger) = \mathrm{Tr}(U_R \Phi^\dagger \Phi U_R^\dagger) = \mathrm{Tr}(\Phi^\dagger \Phi \underbrace{U_R^\dagger U_R}_{=I}) = \mathrm{Tr}(\Phi^\dagger \Phi),\\
\mathrm{Tr}(\tilde{\Phi}'^\dagger\, \Phi') &= \mathrm{Tr}(U_R \tilde{\Phi}^\dagger \underbrace{U_L^\dagger\, U_L}_{=I} \Phi U_R^\dagger) = \mathrm{Tr}(U_R \tilde{\Phi}^\dagger \Phi U_R^\dagger) = \mathrm{Tr}(\tilde{\Phi}^\dagger \Phi \underbrace{U_R^\dagger U_R}_{=I}) = \mathrm{Tr}(\tilde{\Phi}^\dagger \Phi),\\
\chi_L'^\dagger \chi_L' &= \chi_L^\dagger \underbrace{U_L^\dagger\, U_L}_{=I} \chi_L = \chi_L^\dagger \chi_L,\\
\chi_R'^\dagger \chi_R' &= \chi_R^\dagger \underbrace{U_R^\dagger\, U_R}_{=I} \chi_L = \chi_R^\dagger \chi_R,
\end{aligned} \tag{2.43}$$

  hence, the term $V^{(2)}$ is invariant due to the established gauge symmetry.

- From the expression:

$$V^{(4a)} = \frac{1}{2}\sum_{i,j}^{2}\left[\lambda_{ij}\, Tr(\Phi_i^\dagger\, \Phi_j)^2 + \tilde{\lambda}_{ij}\, Tr(\tilde{\Phi}_i^\dagger\, \Phi_j)^2 + H.C.\right]$$

$$\begin{aligned}
\mathrm{Tr}\left(\Phi'^\dagger\, \Phi'\right)^2 &= \mathrm{Tr}\left(\Phi'^\dagger\, \Phi' \Phi'^\dagger\, \Phi'\right) = \mathrm{Tr}\left(U_R \Phi^\dagger \underbrace{U_L^\dagger\, U_L}_{=I} \Phi \underbrace{U_R^\dagger U_R}_{=I} \Phi^\dagger \underbrace{U_L^\dagger\, U_L}_{=I} \Phi U_R^\dagger\right)\\
&= \mathrm{Tr}\left(U_R \Phi^\dagger \Phi \Phi^\dagger \Phi U_R^\dagger\right) = \mathrm{Tr}\left(\Phi^\dagger \Phi \Phi^\dagger \Phi \underbrace{U_R^\dagger U_R}_{=I}\right) = \mathrm{Tr}\left(\Phi^\dagger \Phi \Phi^\dagger \Phi\right) = \mathrm{Tr}\left(\Phi^\dagger\, \Phi\right)^2\\
\mathrm{Tr}\left(\tilde{\Phi}'^\dagger\, \Phi'\right)^2 &= \mathrm{Tr}\left(\tilde{\Phi}'^\dagger\, \Phi' \tilde{\Phi}'^\dagger\, \Phi'\right) = \mathrm{Tr}\left(U_R \tilde{\Phi}^\dagger \underbrace{U_L^\dagger\, U_L}_{=I} \Phi \underbrace{U_R^\dagger U_R}_{=I} \tilde{\Phi}^\dagger \underbrace{U_L^\dagger\, U_L}_{=I} \Phi U_R^\dagger\right)\\
&= \mathrm{Tr}\left(U_R \tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi U_R^\dagger\right) = \mathrm{Tr}\left(\tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi \underbrace{U_R^\dagger U_R}_{=I}\right) = \mathrm{Tr}\left(\tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi\right) = \mathrm{Tr}\left(\tilde{\Phi}^\dagger\, \Phi\right)^2
\end{aligned} \tag{2.44}$$

  hence, the term $V^{(4a)}$ is also invariant due to the established gauge symmetry.

- From the expression:

$$V^{(4b)} = \frac{1}{2}\sum_{i,j}^{2}\left[\lambda'_{ij}\,(Tr\Phi_i^{\dagger}\,\Phi_j)^2 + \tilde{\lambda}'_{ij}\,(Tr\tilde{\Phi}_i^{\dagger}\,\Phi_j)^2 + H.C.\right],$$

however, from the terms of $V^{(2)}$, we have already shown the gauge invariance of the traces:

$$\begin{aligned} \mathrm{Tr}(\Phi'^{\dagger}\,\Phi') &= \mathrm{Tr}(\Phi^{\dagger}\Phi), \\ \mathrm{Tr}(\tilde{\Phi}'^{\dagger}\,\Phi') &= \mathrm{Tr}(\tilde{\Phi}^{\dagger}\Phi), \end{aligned} \tag{2.45}$$

hence, the term $V^{(4b)}$ is also invariant due to the established gauge symmetry.

- From the expression:

$$V^{(4c)} = \rho_{12}\,Tr(\Phi_1^{\dagger}\Phi_1\Phi_2^{\dagger}\Phi_2) + \tilde{\rho}_{12}\,Tr(\tilde{\Phi}_1^{\dagger}\Phi_1\tilde{\Phi}_2^{\dagger}\Phi_2),$$

then

$$\begin{aligned} \mathrm{Tr}(\Phi_1'^{\dagger}\Phi_1'\Phi_2'^{\dagger}\Phi_2') &= \mathrm{Tr}(U_R\Phi_1^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\Phi_1\underbrace{U_R^{\dagger}U_R}_{=I}\Phi_2^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\Phi_2U_R^{\dagger}), \\ &= \mathrm{Tr}(U_R\Phi_1^{\dagger}\Phi_1\Phi_2^{\dagger}\Phi_2U_R^{\dagger}) = \mathrm{Tr}(\Phi_1^{\dagger}\Phi_1\Phi_2^{\dagger}\Phi_2\underbrace{U_R^{\dagger}U_R}_{=I}), \\ &= \mathrm{Tr}(\Phi_1^{\dagger}\Phi_1\Phi_2^{\dagger}\Phi_2), \\ \mathrm{Tr}(\tilde{\Phi}_1'^{\dagger}\Phi_1'\tilde{\Phi}_2'^{\dagger}\Phi_2') &= \mathrm{Tr}(U_R\tilde{\Phi}_1^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\Phi_1\underbrace{U_R^{\dagger}U_R}_{=I}\tilde{\Phi}_2^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\Phi_2U_R^{\dagger}), \\ &= \mathrm{Tr}(U_R\tilde{\Phi}_1^{\dagger}\Phi_1\tilde{\Phi}_2^{\dagger}\Phi_2U_R^{\dagger}) = \mathrm{Tr}(\tilde{\Phi}_1^{\dagger}\Phi_1\tilde{\Phi}_2^{\dagger}\Phi_2\underbrace{U_R^{\dagger}U_R}_{=I}), \\ &= \mathrm{Tr}(\tilde{\Phi}_1^{\dagger}\Phi_1\tilde{\Phi}_2^{\dagger}\Phi_2), \end{aligned} \tag{2.46}$$

hence, the term $V^{(4c)}$ is also invariant due to the established gauge symmetry.

- From the expression:

$$\begin{aligned} V^{(4d)} &= \frac{1}{2}\Bigg[\sum_{i,j}^{2}(\Lambda_{ij}\,Tr\Phi_i^{\dagger}\Phi_j + \tilde{\Lambda}_{ij}\,Tr\tilde{\Phi}_i^{\dagger}\Phi_j)(\chi_L^{\dagger}\chi_L + \chi_R^{\dagger}\chi_R) \\ &+ \bar{\Lambda}_{ij}(\chi_L^{\dagger}\Phi_i\Phi_j^{\dagger}\chi_L + \chi_R^{\dagger}\Phi_i^{\dagger}\Phi_j\chi_R) + \Omega_{ij}(\chi_L^{\dagger}\tilde{\Phi}_i\Phi_j^{\dagger}\chi_L + \chi_R^{\dagger}\tilde{\Phi}_i^{\dagger}\Phi_j\chi_R) \\ &+ \bar{\Lambda}'_{ij}(\chi_L^{\dagger}\tilde{\Phi}_i\tilde{\Phi}_j^{\dagger}\chi_L + \chi_R^{\dagger}\tilde{\Phi}_i^{\dagger}\tilde{\Phi}_j\chi_R) + \Omega'_{ij}(\chi_L^{\dagger}\Phi_i\tilde{\Phi}_j^{\dagger}\chi_L + \chi_R^{\dagger}\Phi_i^{\dagger}\tilde{\Phi}_j\chi_R) + H.C.\Bigg] \end{aligned}$$

The first line has already been demonstrated. From the second line we have:

$$\begin{aligned} \chi_L'^{\dagger}\Phi'\Phi'^{\dagger}\chi_L' &= (U_L\chi_L)^{\dagger}\,U_L\Phi U_R^{\dagger}U_R\Phi^{\dagger}U_L^{\dagger}U_L\chi_L \\ &= \chi_L^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\Phi\underbrace{U_R^{\dagger}U_R}_{=I}\Phi^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\chi_L = \chi_L^{\dagger}\Phi\Phi^{\dagger}\chi_L \end{aligned} \tag{2.47}$$

$$\begin{aligned} \chi_R'^{\dagger}\Phi'^{\dagger}\Phi'\chi_R' &= (U_R\chi_R)^{\dagger}\,U_R\Phi^{\dagger}U_L^{\dagger}U_L\Phi U_R^{\dagger}U_R\chi_R \\ &= \chi_R^{\dagger}\underbrace{U_R^{\dagger}U_R}_{=I}\Phi^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\Phi\underbrace{U_R^{\dagger}U_R}_{=I}\chi_R = \chi_R^{\dagger}\Phi^{\dagger}\Phi\chi_R \end{aligned} \tag{2.48}$$

$$\begin{aligned}
\chi_L'^{\dagger}\tilde{\Phi}'\Phi'^{\dagger}\chi_L' &= (U_L\chi_L)^{\dagger}\, U_L\tilde{\Phi}U_R^{\dagger}U_R\Phi^{\dagger}U_L^{\dagger}U_L\chi_L \\
&= \chi_L^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\,\tilde{\Phi}\,\underbrace{U_R^{\dagger}U_R}_{=I}\,\Phi^{\dagger}\,\underbrace{U_L^{\dagger}U_L}_{=I}\,\chi_L = \chi_L^{\dagger}\tilde{\Phi}\Phi^{\dagger}\chi_L
\end{aligned} \tag{2.49}$$

$$\begin{aligned}
\chi_R'^{\dagger}\tilde{\Phi}'^{\dagger}\Phi'\chi_R' &= (U_R\chi_R)^{\dagger}\, U_R\tilde{\Phi}^{\dagger}U_L^{\dagger}U_L\Phi U_R^{\dagger}U_R\chi_R \\
&= \chi_R^{\dagger}\underbrace{U_R^{\dagger}U_R}_{=I}\,\tilde{\Phi}^{\dagger}\,\underbrace{U_L^{\dagger}U_L}_{=I}\,\Phi\,\underbrace{U_R^{\dagger}U_R}_{=I}\,\chi_R = \chi_R^{\dagger}\tilde{\Phi}^{\dagger}\Phi\chi_R
\end{aligned} \tag{2.50}$$

$$\begin{aligned}
\chi_L'^{\dagger}\tilde{\Phi}'\tilde{\Phi}'^{\dagger}\chi_L' &= (U_L\chi_L)^{\dagger}\, U_L\tilde{\Phi}U_R^{\dagger}U_R\tilde{\Phi}^{\dagger}U_L^{\dagger}U_L\chi_L \\
&= \chi_L^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\,\tilde{\Phi}\,\underbrace{U_R^{\dagger}U_R}_{=I}\,\tilde{\Phi}^{\dagger}\,\underbrace{U_L^{\dagger}U_L}_{=I}\,\chi_L = \chi_L^{\dagger}\tilde{\Phi}\tilde{\Phi}^{\dagger}\chi_L
\end{aligned} \tag{2.51}$$

$$\begin{aligned}
\chi_R'^{\dagger}\tilde{\Phi}'^{\dagger}\tilde{\Phi}'\chi_R' &= (U_R\chi_R)^{\dagger}\, U_R\tilde{\Phi}^{\dagger}U_L^{\dagger}U_L\Phi U_R^{\dagger}U_R\chi_R \\
&= \chi_R^{\dagger}\underbrace{U_R^{\dagger}U_R}_{=I}\,\tilde{\Phi}^{\dagger}\,\underbrace{U_L^{\dagger}U_L}_{=I}\,\tilde{\Phi}\,\underbrace{U_R^{\dagger}U_R}_{=I}\,\chi_R = \chi_R^{\dagger}\tilde{\Phi}^{\dagger}\tilde{\Phi}\chi_R
\end{aligned} \tag{2.52}$$

$$\begin{aligned}
\chi_L'^{\dagger}\Phi'\tilde{\Phi}'^{\dagger}\chi_L' &= (U_L\chi_L)^{\dagger}\, U_L\Phi U_R^{\dagger}U_R\tilde{\Phi}^{\dagger}U_L^{\dagger}U_L\chi_L \\
&= \chi_L^{\dagger}\underbrace{U_L^{\dagger}U_L}_{=I}\,\Phi\,\underbrace{U_R^{\dagger}U_R}_{=I}\,\tilde{\Phi}^{\dagger}\,\underbrace{U_L^{\dagger}U_L}_{=I}\,\chi_L = \chi_L^{\dagger}\Phi\tilde{\Phi}^{\dagger}\chi_L
\end{aligned} \tag{2.53}$$

$$\begin{aligned}
\chi_R'^{\dagger}\Phi'^{\dagger}\tilde{\Phi}'\chi_R' &= (U_R\chi_R)^{\dagger}\, U_R\Phi^{\dagger}U_L^{\dagger}U_L\tilde{\Phi}U_R^{\dagger}U_R\chi_R \\
&= \chi_R^{\dagger}\underbrace{U_R^{\dagger}U_R}_{=I}\,\Phi^{\dagger}\,\underbrace{U_L^{\dagger}U_L}_{=I}\,\tilde{\Phi}\,\underbrace{U_R^{\dagger}U_R}_{=I}\,\chi_R = \chi_R^{\dagger}\Phi^{\dagger}\tilde{\Phi}\chi_R
\end{aligned} \tag{2.54}$$

hence, the term $V^{(4d)}$ is also invariant due to the established gauge symmetry.

- From the expression:

$$V^{(4e)} = \lambda_{LR}\left[(\chi_L^{\dagger}\chi_L)^2 + (\chi_R^{\dagger}\chi_R)^2\right].$$

  the terms of this potential have already been demonstrated in previous calculations its gauge invariance.

Finally, our potential turns out to be invariant by the gauge group of the model. Although we have shown the invariance with respect to the $SU(2)_L \otimes SU(2)_R$ group, in the case of the invariance with respect to $U(1)_{B-L}$ it is automatic for each scalar field of each multiplet.

## 2.4 Gauge Sector: Vectorial Bosons

From the covariant derivative of the scalar sector, for the bi-doublets, $\Phi_i$, $i = 1, 2$ and for the doublets $\chi_L$ and $\chi_R$ are given by:

$$\begin{aligned}
\mathcal{D}_\mu\Phi_i &= \partial_\mu\Phi_i + ig\left[\frac{\vec{\tau}}{2}\cdot\vec{W}_L\Phi_i - \Phi_i\frac{\vec{\tau}}{2}\cdot\vec{W}_R\right], \\
\mathcal{D}_\mu\chi_L &= \left(\partial_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_L - i\,g'B_\mu\right)\chi_L, \\
\mathcal{D}_\mu\chi_R &= \left(\partial_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_R - i\,g'B_\mu\right)\chi_R,
\end{aligned} \tag{2.55}$$

where we have already established $g_L = g_R = g$. Moreover, it is important to recognize that the form of the covariant derivative is justified by the gauge symmetry group of the model, see the ref[30]. The lagrangian density for the kinetic part of the scalar sector is given by:

$$\mathcal{L}_{kinetic} = (\mathcal{D}_\mu \chi_L)^\dagger \mathcal{D}^\mu \chi_L + (\mathcal{D}_\mu \chi_R)^\dagger \mathcal{D}^\mu \chi_R + \mathrm{Tr}\,(\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi, \tag{2.56}$$

replacing the VEVs, expressions (2.14) and (2.15), in the Lagrangian density, (2.56), and separating the quadratic part, we can obtain the mass matrices of the gauge bosons, that will be discussed in detail in the next chapter

## 2.5 Yukawa Sector

As we show above, the model has two bi-doublets, $\Phi_i$, $i = 1, 2$, shown in (2.12), two doublets as $\chi_{L,R}$ shown in (2.13), and no triplets, where leptons interact only with $\Phi_1$ while the quarks with $\Phi_2$.

To build the Yukawa sector we are going to introduce the following $Z_2 \otimes \mathbb{Z}_2'$ discrete symmetry (direct product of two Cyclic group $\mathbb{Z}_N$). In the next chapter this symmetry will be apply to the scalar potential of our model in order to make it easier to work. On the other hand, under the first one, $Z_2$, right-handed lepton doublets, $R'$, and $\Phi_1$ are odd, that is, $R'$, $\Phi_1 \to -R'$, $-\Phi_1$; while other fields are even; under the second one, $\mathbb{Z}_2'$, right-handed quarks $Q_R$ and $\Phi_2$ is odd ($Q'_R$, $\Phi_2 \to -Q'_R$, $-\Phi_2$). This is the justification for why the $\Phi_1$ bi-doublet couples with the leptons while the $\Phi_2$ bi-doublet couples with the quarks.

Hence, in this case the Yukawa interactions in the lepton sector are:

$$-\mathcal{L}_Y = \bar{L}'(G\Phi_1 + F\tilde{\Phi}_1)R' + \bar{R}'(\Phi_1^\dagger G^\dagger + \tilde{\Phi}_1^\dagger F^\dagger)L' \tag{2.57}$$

where $L'$ and $R'$ are defined in (2.1) and we have omitted generations indices. To observe that the terms $\chi_L^\dagger \Phi_{1,2} \chi_R$ (trilinear term) in the scalar potential are forbided under these symmetries.

Imposing left-right symmetry to this Yukawa's sector, that is: $L' \leftrightarrow R'$, $\Phi_i \leftrightarrow \Phi_i^\dagger$, $\tilde{\Phi}_i \leftrightarrow \tilde{\Phi}_i^\dagger$, we obtain:

$$-\mathcal{L}_Y = \bar{R}'(G\Phi_1^\dagger + F\tilde{\Phi}_1^\dagger)L' + \bar{L}'(\Phi_1 G^\dagger + \tilde{\Phi}_1 F^\dagger)R' \tag{2.58}$$

comparing both lagrangian density, (2.57) and (2.58), implies that $G^\dagger = G$ and $F^\dagger = F$. Hence these matrices can be diagonalized by unitary transformations. A similar expression arises in the quarks sector but now $\Phi_1 \to \Phi_2$ and $(\nu'_{L,R}, \ell'_{L,R}) \to (u'_{L,R}, d'_{L,R})$. Primed fields denote symmetry eigenstates and unprimed ones mass eigenstates.

With these interactions the mass matrices in the lepton sector are:

$$M^\nu = G\frac{k_1}{\sqrt{2}} + F\frac{k_1'^*}{\sqrt{2}}, \quad M^l = G\frac{k_1'}{\sqrt{2}} + F\frac{k_1^*}{\sqrt{2}}. \tag{2.59}$$

In general $G$, $F$ are hermitians and the VEVs are complex, then the mass matrices are diagonalized by biunitary transformations as follows:

$$V_L^{l\dagger} M^l V_R^l = \hat{M}^l, \quad U_L^{\nu\dagger} M^\nu U_R^\nu = \hat{M}^\nu, \tag{2.60}$$

where $\hat{M}^l = \mathrm{diag}(m_e, m_\mu, m_\tau)$ and $\hat{M}^\nu = \mathrm{diag}(m_1, m_2, m_3)$ for charged leptons and neutrinos respectively.

To give an appropriate mass to the quarks, we have to introduce the bidoublet $\Phi_2$, and it is possible to implement the analysis as in Ref. [31]. Notice that this means that the neutral scalar with VEV and mass about 174 and 125 GeV, respectively, is part of this bidoublet.

We will assume that $k'_1 = 0$ (for simplicity) and in this case the lepton mass matrices, from (2.59), are given by

$$M^{\nu}_{ab} = G_{ab}\frac{k_1}{\sqrt{2}}, \quad M^{l}_{ab} = F_{ab}\frac{k_1^*}{\sqrt{2}}, \tag{2.61}$$

where $G$ and $F$ are symmetric complex matrices that are diagonalized, in general, by the biunitary transformation in Eq. (2.60). However, we will consider, just for the sake of simplicity, all VEVs being real too. From these matrices and the lepton measured masses, we found the Yukawa coupling matrices

$$G = \frac{\sqrt{2}}{k_1}U^{\nu}_L \hat{M}^{\nu} U^{\nu\dagger}_R, \quad F = \frac{\sqrt{2}}{k_1^*}V^l_L \hat{M}^l V^{l\dagger}_R, \tag{2.62}$$

and we use for numerical calculations $|k_1| = 2$ GeV, since this VEV is the only one for generating the lepton masses. Then, making uso of (2.61), we can regard $U^{\nu}_L = U^{\nu}_R \equiv U^{\nu}$, and $U^{\nu} = V^L_{PMNS} = V^R_{PMNS} \equiv V_l$, and we have

$$G = \frac{\sqrt{2}}{k_1}V_l \hat{M}^{\nu} V_l^{\dagger}, \quad F = \frac{\sqrt{2}}{k_1}\hat{M}^l, \tag{2.63}$$

the unitary matrix $V_l$ being parametrized in the same way for Dirac particles. We use the PDG parametrization for Dirac neutrinos, for the interactions with $W^+_{L,R}$:

$$V_l = \begin{pmatrix} c^l_{12}c^l_{13} & s^l_{12}c^l_{13} & s^l_{13} \\ -s^l_{12}c^l_{23} - c^l_{12}s^l_{13}s^l_{23} & c^l_{12}c^l_{23} - s^l_{12}s^l_{13}s^l_{23} & c^l_{13}s^l_{23} \\ s^l_{12}s^l_{23} - c^l_{12}s^l_{13}c^l_{23} & -c^l_{12}s^l_{23} - s^l_{12}s^l_{13}c^l_{23} & c^l_{13}c^l_{23} \end{pmatrix}, \tag{2.64}$$

with $s^l_{ij} = \sin\theta^l_{ij}, \cdots$ and where we have considered $\delta_l = 0$.

In this case the Yukawa interactions are given by

$$\begin{aligned} -\mathcal{L}^Y_l &= \frac{\sqrt{2}}{k_1}\{\bar{\nu}_L[(\hat{M}^{\nu}\phi^0_1 + V_l^{\dagger}\hat{M}^l V_l \eta^{0*}_1)\nu_R + (\hat{M}^{\nu}V_l^{\dagger}\eta^+_1 - V_l^{\dagger}\hat{M}^l\phi^+_1)l_R] \\ &+ \bar{l}_L[(V_l\hat{M}^{\nu}\phi^-_1 - \hat{M}^l V_l \eta^-_1)\nu_R + (V_l\hat{M}^{\nu}V_l^{\dagger}\eta^0_1 + \hat{M}^l\phi^{0*}_1)l_R]\} \\ &+ H.c., \end{aligned} \tag{2.65}$$

with $V_l$ given in (2.64). Notice that, in this case (in the basis in which charged leptons are diagonal), the Higgs $\phi^0_1$ is the one whose couplings with charged leptons are proportional to their respective masses, and the couplings with $\eta^0_1$ are suppressed by the neutrino masses in the charged lepton sector. In the neutrino sector, the situation reverses: The enhanced interactions are those with $\eta^0_1$ since they are proportional to the charged lepton masses.

In addition, we write the Yukawa interactions in the quark sector (with their mass matrices diagonalized by the unitary matrices $V^u_{L,R}$ and $V^d_{L,R}$ with $V^L_{CKM} = V^R_{CKM} = V^{u\dagger}_L V^d_L$):

$$\begin{aligned} -\mathcal{L}^Y_q &= \frac{\sqrt{2}}{k_2}\{\bar{u}_L V^{u\dagger}[(G_q V^u\phi^0_2 + F_q\eta^{0*}_2)V^u\, u_R + (G_q\eta^+_2 - F_q\phi^+_2)V^d d_R] \\ &+ \bar{d}_L V^{d\dagger}[(G_q\phi^-_2 - F_q\eta^-_2)V^u\, u_R + (G_q\eta^0_2 + F_q\phi^{0*}_2)V^d d_R]\} + H.c. \end{aligned} \tag{2.66}$$

The solution $k_2' = 0$ should not regard in the quark sector since for the case of generalized parity, $\mathcal{P}$, it has been shown that $k_2' \ll k_2$ is ruled out by the $CP$-violating parameters $\epsilon$ and $\epsilon'$; nevertheles, in the case of generalized $\mathcal{C}$ this hierarchy is allowed [32].

Notice that there are flavor-changing neutral currents (FCNCs) mediated by scalars in both lepton and quark sectors. However, the existence of these processes in the present model implies only that the constraints already obtained in the minimal version of the model i.e., one bidoublet and two doublets, have to be reviewed. For instance, the current data and the contributions related to the renormalization of the flavor-changing neutral Higgs tree-level amplitude which are needed in order to obtain gauge-independent results was done in Ref. [32]. In the minimal model there are four neutral scalars while in the present model there are six (with two bidoublets) or eight (three bidoublets) and it means that there are more amplitude mediated by neutral scalars at tree and one-loop level than those in Ref. [32]. Doing this analysis is outside the scope of this paper.

## 2.6 Alternatives models relating to the $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ gauge symmetry

### 2.6.1 A model for Dirac Neutrino masses and mixings - Rabindra N. Mohapatra

In this article[33], Mohapatra present a gauge model for three-fermion families where the neutrino masses vanish naturally at the tree level and their Dirac masses arise at the one-loop level.

Mohapatra regard the following gauge group $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ in order to describe the electroweak interactions for $E \gg M_W$ with quark and lepton doublets assigned in the ussual way:

$$\text{Quarks: } Q_L(1/2,0,1/3),\ Q_R(0,1/2,1/3);\ \text{ leptons: } \psi_L(1/2,0,-1),\ \psi_R(0,1/2,-1), \quad (2.67)$$

where: $Q \equiv (u,d)$ and $\psi \equiv (\nu, e)$ and so on for higher generations.

We can find two stages:

1. In the first, this model describe the symmetries of the model which allow us to preserve the neutrinos naturally massless at the tree level.
2. In the second, the author computes corrections to one loop neutrino Dirac masses, regarding that the lepton number is a good symmetry de the model. The neutrinos will be Dirac particles and the eigenvalues and eigenvectors of this one loop mass matrix provide the physical masses and mixings betweeen the different neutrino species.

The following symmetries are imposed on the model, $(D_q \times D_\ell)$:

$$\begin{aligned} D_q: \quad & Q_L \to -Q_L, \quad Q_R \to iQ_R, \quad \Phi_q \to i\Phi_q, \\ & \chi_L \to \omega\chi_L, \quad \chi_R \to -i\chi_R, \\ & \sigma_1 \to +i\omega\sigma_1, \quad \sigma_2 \to \omega\sigma_2, \quad \sigma_1 \to \omega\sigma_3, \end{aligned} \quad (2.68)$$

where $\omega = \exp(\frac{2}{3}\pi i)$, and others symmetries, like:

$$\begin{array}{lll} D_\ell : & \psi_L \to i\exp(\frac{1}{4}i\pi)\,\psi_L & \psi_R \to i\exp(\frac{1}{4}i\pi)\psi_R \\ & \Phi_\ell \to i\Phi, & \eta_{L1} \to i\eta_{L1}, \;\; \eta_{R1} \to i\eta_{R1} \\ & \sigma_2 \to +\sigma_2 & \sigma_3 \to -i\sigma_3. \end{array} \tag{2.69}$$

The Yukawa couplings invariant under $D_\ell \times D_q$ are:

$$\begin{aligned} \mathcal{L}_Y &= h_1\bar{Q}_L\Phi_q Q_R + h_2(\bar{Q}_L\chi_L g_{R1} + \bar{Q}_R\chi_R g_{L1}) + \mu_1(\bar{g}_{L1}g_{R2} + \bar{g}_{L2}g_{R1}) + \\ &+ h_3\bar{g}_{L2}g_{R2}\sigma_1 + h_4\bar{\psi}_L\Phi_\ell\psi_R + f(\psi_L^T\tau_2 C^{-1}\psi_L\eta_{L1} + \psi_R^T\tau_2 C^{-1}\psi_R\eta_{R1}) + h.c. \end{aligned} \tag{2.70}$$

Where $h_i$ $(i = 1, 2, 3, 4)$ and $\mu 1$ are matrices. By choosing a proper basis for $Q_{L,R}$ and $\psi_{L,R}$ it is posible to diaginalize $h_{1,4}$, $h_3$. The matrices $h_2$ and $\mu_1$ are the sole source of flavor mixing in the quark sector.

Mohapatra calculates the one loop contribution to the neutrino mass matrix. They arise from the exchange of the $\eta_L$ and $\eta_R$ bosons in the loop (the loop is shown in the Mohapatra's article) and has the form:

$$(M_\nu)_{ab} = (f^T M_\ell \epsilon f)_{ab}, \tag{2.71}$$

where:

$$\epsilon_a \approx (1/16\pi^2)\ln(M_\eta^2/m_a^2), \tag{2.72}$$

where $a$ denotes the leptonic generation; $\epsilon$ is a diagonal matrix with diagonal elements: $Diag\,M = (m_2, m_\mu, m_\tau)$.

In conclusion, Mohapatra presents a model for Dirac neutrino masses and mixings, $m_{\nu_\mu} \gg m_{\nu_\tau} \gg m_{\nu_e}$, with all mixings given in terms of two parameters. This model can also provide a solution to the solar neutrino puzzle.

### 2.6.2 Neutrino Mass and Spontaneous Parity Nonconservation - Rabindra N. Mohapatra and Goran Senjanovic

In this article[34], they propose a model of spontaneous parity nonconservation based on the $SU(2)_R \otimes SU(2)_L \otimes U(1)_{Y'}$ gauge group, where this connection is brought out explicitly. It is obtained the following estimate that relates the neutrino mass to the mass of the right-handed gauge bosons:

$$m_{\nu_e} \simeq m_e^2/gm_{W_R}.$$

Where a similar formula holds for leptons in each generation.

The main new ingredient of this proposal is that they start with two Majorana neutrinos $\nu$ and $N$ and choose the left- and right-handed lepton multiplets prior to spontaneous breakdown to be:

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \psi_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \tag{2.73}$$

They impose the left-right symmetry on the Lagrangian; under this symmetry $\varphi_L \leftrightarrow \varphi_R$ and this demand that at three level, $g_L = g_R$.

They also introduce the Higgs multiplets to break the gauge symmetry down to $U(1)_{em}$: One bi-doublet and two triplets that transform like: $\varphi \sim (\frac{1}{2}, \frac{1}{2}, 0)$, $\Delta_L \sim (1, 0, 2)$, and $\Delta_R \sim (0, 1, 2)$.

The Yukawa sector can be written as:

$$\mathcal{L} = h_1 \bar{\psi}_L \varphi \psi_R + h_2 \psi_L \tilde{\varphi} \psi_R + h_3(\psi_L^T C i \tau_2 \Delta_L \psi_L + \psi_R^T C i \tau_2 \Delta_R \psi_R) + H.C., \tag{2.74}$$

where: $\tilde{\varphi} \equiv \tau_2 \varphi^* \tau_2$ and C is the Dirac charge-conjugation matrix. In addition:

$$\Delta_{L,R} = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta^+ & \delta^{++} \\ \delta^+ & -\frac{1}{\sqrt{2}}\delta^+ \end{pmatrix}_{L,R} \tag{2.75}$$

Under left-right discrete symmetry:

$$\varphi \leftrightarrow \varphi^\dagger \qquad \Delta_L \leftrightarrow \Delta_R$$

With the analysis of charged and neutral current phenomena puts a lower bound on the right-handed W-boson mass, that is:

$$m_{W_R} \geq 250 \; - \; 300 \;\; \text{GeV} \tag{2.76}$$

If it is chosen $m_{W_R} \geq 300 \; GeV$ then it will produce:

$$m_{\nu_e} \leq 1.5 \; eV, \;\; m_{\nu_\mu} < 56 \; keV, \;\; m_{\nu_\tau} \leq 18 \; MeV \tag{2.77}$$

This model provides, therefore, an understanding of a tiny neutrino mass.

# Chapter 3

# Characteristic of the Model with Gauge Symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes \mathcal{P}$

## 3.1 Scalar sector - Reduced Scalar Potential

Taking into account the expression of the most general scalar potential given in the equation (2.17), we are going to apply certain discrete symmetries to obtain a simpler version of the scalar potential.

### 3.1.1 Imposing some discrete symmetries to our scalar potential

Two constrain equations are shown below, the rest of equations can be found in the corresponding appendix. In addition, to calculate the constrain equations we carry out the following operation:

$$t_i = \frac{\partial V}{\partial X_i} = 0,$$

where $X_i = k_1, k_1', k_2, k_2', v_L, v_R$ (we have considered real VEVs because we are not interesting to study CP Violation), therefore, they are given by:

$$\begin{aligned} t_1 &= k_1 \Big[ \mu_{11}^2 + (\lambda_{11} + \lambda_{11}')\, k_1^2 + k_1'^2 \left( \lambda_{11}' + \tilde{\lambda}_{11} + 2\tilde{\lambda}_{11}' \right) + \frac{1}{2} \Big( v_R^2 H + \tilde{\lambda}_{21} k_2'^2 \\ &+ \left( \tilde{\lambda}_{12} + \tilde{\lambda}_{21}' + \tilde{\lambda}_{12}' \right) k_2'^2 + \frac{1}{2} v_L^2 H + k_2^2 \left( \lambda_{12}' + \lambda_{21}' + \lambda_{21} + \lambda_{12} + \rho_{12} \right) \Big) \Big] \\ &+ \frac{1}{4}(v_R^2 + v_L^2)(k_1' D + k_2' F + k_2 G) + \frac{k_2 k_1' k_2'}{2} \left( \tilde{\lambda}_{21}' + \tilde{\lambda}_{12}' + \lambda_{12}' + \lambda_{21}' + \tilde{\rho}_{12} \right) \\ &+ k_2 \mu^2 + \tilde{\mu}_{11}^2 k_1', \end{aligned} \tag{3.1}$$

$$\begin{aligned} t_1' &= k_1' \Big[ \mu_{11}^2 + (\lambda_{11} + \lambda_{11}')\, k_1'^2 + k_1^2 \left( \lambda_{11}' + \tilde{\lambda}_{11} + 2\, \tilde{\lambda}_{11}' \right) + \frac{1}{2} \Big( v_R^2 H + \tilde{\lambda}_{12}' k_2^2 \\ &+ (\tilde{\lambda}_{21}' + \tilde{\lambda}_{12} + \tilde{\lambda}_{21}) k_2^2 + \frac{1}{2} v_L^2 H + k_2'^2 \left( \lambda_{12}' + \lambda_{21}' + \lambda_{21} + \lambda_{12} + \rho_{12} \right) \Big) \Big] \\ &+ \frac{1}{4} \left( v_L^2 + v_R^2 \right) (k_1 D + k_2 F + k_2' G) + \frac{k_2 k_1 k_2'}{2} \left( \tilde{\lambda}_{12}' + \tilde{\lambda}_{21}' + \lambda_{12}' + \lambda_{21}' + \tilde{\rho}_{12} \right) \\ &+ k_2' \mu^2 + \tilde{\mu}_{11}^2 k_1, \end{aligned} \tag{3.2}$$

and similarly we obtain $t_2$ and $t_2'$ for $k_2$ and $k_2'$, respectively, but we will not write them explicitly. Finally, we have

$$t_L = \frac{v_L}{2}(2\mu_{LR}^2 + 2\lambda_{LR}v_L^2 + \Delta), \quad t_R = \frac{v_R}{2}(2\mu_{LR}^2 + 2\lambda_{LR}v_R^2 + \Delta), \tag{3.3}$$

where

$$\begin{aligned} \Delta &= k_1'^2 A + k_1' k_2' B + k_1' k_2 C + k_1 k_1' D + k_2 k_2' E + k_1 k_2' F + k_1 k_2 G + k_1^2 H + k_2^2 I \\ &+ k_2'^2 J, \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} A &= \Lambda_{11} + \overline{\Lambda}_{11},\ B = \Lambda_{12} + \Lambda_{21} + \overline{\Lambda}_{12} + \overline{\Lambda}_{21},\ C = \tilde{\Lambda}_{12} + \tilde{\Lambda}_{21} + \Omega_{12}', +\Omega_{21} \\ D &= \Omega_{11}' + 2\,\tilde{\Lambda}_{11} + \Omega_{11},\ E = \Omega_{22}' + 2\,\tilde{\Lambda}_{22} + \Omega_{22},\ F = \Omega_{21}' + \tilde{\Lambda}_{21} + \tilde{\Lambda}_{12} + \Omega_{12}, \\ G &= \overline{\Lambda}_{21}' + \overline{\Lambda}_{12}' + \Lambda_{21} + \Lambda_{12},\ H = \Lambda_{11} + \overline{\Lambda}_{11}', \\ I &= \Lambda_{22} + \overline{\Lambda}_{22}',\ J = \Lambda_{22} + \overline{\Lambda}_{22}, \end{aligned} \tag{3.5}$$

The invariance under the parity transformations defined in (2.5) implies $\mu_{12} = \mu_{21} \equiv \mu^2$, $\tilde{\mu}_{12} = \tilde{\mu}_{21} \equiv \nu^2$; $\lambda_{12} = \lambda_{21},\ \tilde{\lambda}_{12} = \tilde{\lambda}_{21},\ \tilde{\lambda}_{12}' = \tilde{\lambda}_{21}',\ \lambda_{12}' = \lambda_{21}',\ \Lambda_{12} = \Lambda_{21},\ \tilde{\Lambda}_{12} = \tilde{\Lambda}_{21},\ \bar{\Lambda}_{12} = \bar{\Lambda}_{21},\ \bar{\Lambda}_{12}' = \bar{\Lambda}_{21}'$, and that $\Omega_{ij}'$ and $\Omega_{ij}$ are real.

Notice that the $\mathbb{Z}_2 \otimes \mathbb{Z}_2'$ (see the section 2.5) implies $\mu^2 = \nu^2 = 0$ and $\Lambda_{12} = \Lambda_{21} = \bar{\Lambda}_{12} = \bar{\Lambda}_{21} = \bar{\Lambda}_{12}' = \bar{\Lambda}_{21}' = 0$. However, we will allow for the moment a soft breaking of these symmetries and use $\mu^2 \neq 0$ in order to get the hierarchy between the *vevs*.

Notice that only $v_L$ and $v_R$ can be zero; however, this solution is not accepted for $v_R$. Thus, we obtain the hierarchy relation between the *vevs*:

$$v_R \gg k_2 \gg k_1, k_1', k_2' \gg v_L,$$

we are assuming that $v_R$ is very large compare with the others ones, and $v_L$ is the smallest one, this would not affect our results, and If

$$D, F, G \ll 1, \quad \tilde{\lambda}_{12}' + \tilde{\lambda}_{21}' + \lambda_{21}' + \lambda_{12}' + \tilde{\rho}_{12} \ll 1, \tag{3.6}$$

then we obtain from Eqs. (3.1) and (3.2), respectively,

$$k_1 \approx \frac{-\mu^2}{\mu_{11}^2 + v_R^2 H}\, k_2 \ll k_2, \qquad k_1' \approx \frac{-\mu^2}{\mu_{11}^2 + v_R^2 H}\, k_2' \ll k_2', \tag{3.7}$$

with $-\mu^2 > 0$ and $v_R^2 H > |\mu_{11}^2|$. This shows that there is a range of the parameter space in which we can have $k_1' \ll k_1 \ll k_2' < k_2$. Moreover, if we can assume for simplicity:

$$D = F = G = 0, \quad \lambda_{ij} = \lambda_{ij}' = \tilde{\lambda}_{ij} = \tilde{\lambda}_{ij}' = \tilde{\rho}_{ij} = 0,\ i \neq j;\ \mu^2 = \tilde{\mu}_{11}^2 = 0, \tag{3.8}$$

the constraint equations become

$$
\begin{aligned}
t_1 &= k_1\left(\mu_{11}^2 + (\lambda_{11}+\lambda'_{11})k_1^2 + \lambda'_{11}k_1'^2 + \frac{1}{2}(v_L^2+v_R^2)H + \frac{1}{2}\rho_{12}k_2^2\right), \\
t'_1 &= k'_1\left(\mu_{11}^2 + \lambda'_{11}k_1^2 + (\lambda_{11}+\lambda'^2_{11})k_1'^2 + \frac{1}{2}(v_L^2+v_R^2)A + \frac{1}{2}\rho_{12}k_2'^2\right), \\
t_2 &= k_2\left(\mu_{22}^2 + (\lambda_{22}+\lambda'_{22})k_2^2 + \lambda'_{22}k_2'^2 + \frac{1}{2}(v_L^2+v_R^2)I + \frac{1}{2}\rho_{12}k_1^2\right), \\
t'_2 &= k'_2\left(\mu_{22}^2 + (\lambda_{22}+\lambda'_{22})k_2'^2 + \lambda'_{22}k_2^2 + \frac{1}{2}(v_L^2+v_R^2)J + \frac{1}{2}\rho_{12}k_1'^2\right), \\
t_L &= \frac{v_L}{2}\left[2\mu_{LR}^2 + 2\lambda_{LR}v_L^2 + k_1'^2 A + k_1^2 H + k_2^2 I + k_2'^2 J\right], \\
t_R &= \frac{v_R}{2}\left[2\mu_{LR}^2 + 2\lambda_{LR}v_R^2 + k_1'^2 A + k_1^2 H + k_2^2 I + k_2'^2 J\right].
\end{aligned} \tag{3.9}
$$

In fact, we further restrict the Higgs potential so that it is invariant under the $\mathbb{Z}_5$ symmetry (defined as $\omega_n = e^{2\pi\, i\, n/5}$, $n = 0, \cdots, 4$) under which

$$\Phi_1 \to \omega_1\,\Phi_1 = e^{2\pi\, i/5}\,\Phi_1,\;\; \Phi_2 \to \omega_2\,\Phi_2 = e^{4\pi\, i/5}\,\Phi_2$$

while also other fields are invariant; that is:

$$
\begin{aligned}
\mathrm{Tr}(\Phi_1'^\dagger\Phi_2') &= \mathrm{Tr}(\omega_1^*\Phi_1^\dagger\omega_2\Phi_2) = \mathrm{Tr}(\omega_1^*\omega_2\Phi_1^\dagger\Phi_2) = \mathrm{Tr}(e^{2\pi\, i/5}\,\Phi_1^\dagger\Phi_2) = e^{2\pi\, i/5}\,\mathrm{Tr}(\Phi_1^\dagger\Phi_2), \\
\mathrm{Tr}\left(\Phi_1'^\dagger\Phi_2'\right)^2 &= \mathrm{Tr}\left(\omega_1^*\Phi_1^\dagger\omega_2\Phi_2\right)^2 = \mathrm{Tr}\left(e^{2\pi\, i/5}\,\Phi_1^\dagger\Phi_2\right)^2 = e^{4\pi\, i/5}\,\mathrm{Tr}\left(\Phi_1^\dagger\Phi_2\right)^2,
\end{aligned}
$$

when $i \neq j$ the term in the lagrangian is not invariant. Only the terms where i = j remain.

In this way, the scalar potential in Eq. (2.17) becomes

$$
\begin{aligned}
V^{(2)} &= \frac{1}{2}\sum_{i=1,2}\left[\mu_{ii}^2\mathrm{Tr}(\Phi_i^\dagger\Phi_i) + H.c.\right] + \mu_{LR}^2(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R), \\
V^{(4a)} &= \frac{1}{2}\sum_{i=1,2}\left[\lambda_{ii}\mathrm{Tr}(\Phi_i^\dagger\Phi_i)^2 + H.c.\right], \\
V^{(4b)} &= \frac{1}{2}\sum_{i=1,2}\lambda'_{ii}(\mathrm{Tr}\Phi_i^\dagger\Phi_i)^2, \\
V^{(4c)} &= \rho_{12}\mathrm{Tr}(\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2), \\
V^{(4d)} &= \frac{1}{2}\left[\sum_{i=1,2}\left\{\Lambda_{ii}\mathrm{Tr}\Phi_i^\dagger\Phi_i(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R) + \bar{\Lambda}_{ii}(\chi_L^\dagger\Phi_i\Phi_i^\dagger\chi_L + \chi_R^\dagger\Phi_i\Phi_i^\dagger\chi_R) + \right.\right. \\
&+ \left.\left. \bar{\Lambda}'_{ii}(\chi_L^\dagger\tilde{\Phi}_i\tilde{\Phi}_i^\dagger\chi_L + \chi_R^\dagger\tilde{\Phi}_i\tilde{\Phi}_i^\dagger\chi_R)\right\}\right], \\
V^{(4e)} &= \lambda_{LR}[(\chi_L^\dagger\chi_L)^2 + (\chi_R^\dagger\chi_R)^2],
\end{aligned} \tag{3.10}
$$

When the neutral fields gain an expectation value, the minimum value of the potential is obtained; moreover, their constraint equations are also obtained. Theses results are shown below:

- **The minimum value of the new scalar potential:**

$$
\begin{aligned}
\langle V^{(2)}\rangle &= \mu_{LR}^2\left(\frac{v_L^2}{2}+\frac{v_R^2}{2}\right)+\mu_{11}^2\left(\frac{k_1^2}{2}+\frac{k_1'^2}{2}\right)+\mu_{22}^2\left(\frac{k_2^2}{2}+\frac{k_2'^2}{2}\right)\\
\langle V^{(4a)}\rangle &= \lambda_{11}\left(\frac{k_1^4}{4}+\frac{k_1'^4}{4}\right)+\lambda_{22}\left(\frac{k_2^4}{4}+\frac{k_2'^4}{4}\right)\\
\langle V^{(4b)}\rangle &= \lambda_{11}'\left(\frac{k_1^2}{2}+\frac{k_1'^2}{2}\right)^2+\lambda_{22}'\cdot\left(\frac{k_2^2}{2}+\frac{k_2'^2}{2}\right)^2\\
\langle V^{(4c)}\rangle &= \rho_{12}\left(\frac{k_1'^2k_2'^2}{4}+\frac{k_1^2k_2^2}{4}\right)\\
\langle V^{(4d)}\rangle &= \frac{1}{4}(v_L^2+v_R^2)\left(\overline{\Lambda}_{11}\,k_1'^2+\overline{\Lambda}_{22}\,k_2'^2+\overline{\Lambda}_{11}'k_1^2+\overline{\Lambda}_{22}'k_2^2\right)\\
\langle V^{(4e)}\rangle &= \lambda_{LR}\left(\frac{v_R^4}{4}+\frac{v_L^4}{4}\right).
\end{aligned}
\tag{3.11}
$$

- **The new constraint equations become the following:**

$(a)\ k_1:$

$$t_1 = k_1\left(\mu_{11}^2+(\lambda_{11}+\lambda_{11}')k_1^2+\lambda_{11}'k_1'^2+\frac{1}{2}(v_L^2+v_R^2)(\Lambda_{11}+\overline{\Lambda}_{11}')+\frac{1}{2}k_2^2\rho_{12}\right),$$

$(b)\ k_1':$

$$t_1' = k_1'\left(\mu_{11}^2+(\lambda_{11}+\lambda_{11}')k_1'^2+\lambda_{11}'k_1^2+\frac{1}{2}(v_L^2+v_R^2)(\Lambda_{11}+\overline{\Lambda}_{11})+\frac{1}{2}k_2'^2\rho_{12}\right),$$

$(c)\ k_2:$

$$t_2 = k_2\left(\mu_{22}^2+(\lambda_{22}+\lambda_{22}')k_2^2+\lambda_{22}'k_2'^2+\frac{1}{2}(v_L^2+v_R^2)(\Lambda_{22}+\overline{\Lambda}_{22}')+\frac{1}{2}k_1^2\rho_{12}\right),$$

$(d)\ k_2':$

$$t_2' = k_2'\left(\mu_{22}^2+(\lambda_{22}+\lambda_{22}')k_2'^2+\lambda_{22}'k_2^2+\frac{1}{2}(v_L^2+v_R^2)(\Lambda_{22}+\overline{\Lambda}_{22})+\frac{1}{2}k_1'^2\rho_{12}\right),$$

$(e)\ v_L:$

$$
\begin{aligned}
t_L &= v_L\left(\mu_{LR}^2+\lambda_{LR}v_L^2+\frac{1}{2}\left(k_1'^2(\Lambda_{11}+\overline{\Lambda}_{11})+k_2'^2(\Lambda_{22}+\overline{\Lambda}_{22})+k_1^2(\Lambda_{11}+\overline{\Lambda}_{11}')+\right.\right.\\
&+\left.\left. k_2^2(\Lambda_{22}+\overline{\Lambda}_{22}')\right)\right),
\end{aligned}
$$

$(f)\ v_R:$

$$
\begin{aligned}
t_R &= v_R\left(\mu_{LR}^2+\lambda_{LR}v_R^2+\frac{1}{2}\left(k_1'^2(\Lambda_{11}+\overline{\Lambda}_{11})+k_2'^2(\Lambda_{22}+\overline{\Lambda}_{22})+k_1^2(\Lambda_{11}+\overline{\Lambda}_{11}')+\right.\right.\\
&+\left.\left. k_2^2(\Lambda_{22}+\overline{\Lambda}_{22}')\right)\right).
\end{aligned}
\tag{3.12}
$$

We can observe from these constrain equations that are the same as those shown in Eq. (3.9). It means that these conditions are protected by the $\mathbb{Z}_5$ symmetry and may be naturally small. We may consider the potential in Eq. (3.10), and the respective mass spectra, as a good approximation.

Moreover, from the constrain equations, all VEVs could be zero; in particular, the solutions $k'_{1,2} = 0$ and $v_L = 0$ are allowed. The SM-like Higgs scalar is in the bidoublet $\Phi_2$.

It is important to note that the doublet $\chi_L$ was introduced just to implement the invariance of the Lagrangian under parity without interacting with the fermions, if the respective VEV is zero it becomes an inert doublet since the left-right symmetry protects its inert character; hence it is a candidate for dark matter.

Notice that $v_L \neq 0$ is also a solution, hence the possibility to have a model without any bidoublet, with fermion masses arisen from nonrenormalizable interactions [35]; in this case, it is possible to make $A = H = I = J = 0$ in Eq. (3.9). We stress that although the constraint equations in Eq. (3.9) were obtained using the potential in Eq. (3.10) by considering the most general potential (without the $\mathbb{Z}_2 \otimes \mathbb{Z}'_2$ symmetries), we still obtain

$$t_L = v_L(\mu_{LR}^2 + \lambda_{LR} v_L^2 + ...), \tag{3.13}$$

and the solution $v_L = 0$ is still allowed even without a soft breaking of parity symmetry [36].

## 3.2 Gauge boson mass matrix:

In this section, using the covariant derivatives given in equation (2.55) and the lagrangian density (2.56), we proceeded to calculate the masses of the gauge bosons, both for $Z_1$ and $Z_2$ as a function of the VEVs and others parameters proposed in the model; for the case of the photon was obtained an eigenvalue exactly zero, as it should be. Subsequently, using the mass hierarchy of expectation values where $v_R \gg X$, being $X$ : others VeVs different from $v_R$. In this section we have obtained explicit values of the masses of the vector bosons and a lower bound for their right handed vector boson's masses. The couplings of physical bosons with known fermions have also been obtained.

### 3.2.1 Charged Bosons mass matrix:

$$\mathcal{M}_{CB}^2 = \frac{g^2}{4}\begin{pmatrix} k_1^2 + k_1'^2 + k_2^2 + k_2'^2 + v_L^2 & -2(k_1k_1' + k_2k_2') \\ -2(k_1k_1' + k_2k_2') & k_1^2 + k_1'^2 + k_2^2 + k_2'^2 + v_R^2 \end{pmatrix}$$

Diagonalization of the matrix:

$$\det|M - \lambda I| = 0,$$

where: $\mathcal{M}_{CB}^2 = \frac{g^2}{4} M, \quad \lambda$ : eigenvalue of $M$

$$\det|M - \lambda I| = \begin{vmatrix} k_1^2 + k_1'^2 + k_2^2 + k_2'^2 + v_L^2 - \lambda & -2(k_1k_1' + k_2k_2') \\ -2(k_1k_1' + k_2k_2') & k_1^2 + k_1'^2 + k_2^2 + k_2'^2 + v_R^2 - \lambda \end{vmatrix} = 0,$$

in this case we have the characteristic equation for $\lambda$:

$$\lambda^2 - [v_L^2 + v_R^2 + 2K^2]\,\lambda + (K^2 + v_L^2)(K^2 + v_R^2) - 4(k_1k_1' + k_2k_2')^2 \;=\; 0,$$

where: $\quad K^2 \;\equiv\; k_1^2 + k_1'^2 + k_2^2 + k_2'^2$

solving the equation we have:

$$\lambda \;=\; \frac{1}{2}\left(2K^2+v_L^2+v_R^2 \pm \sqrt{(2K^2+v_L^2+v_R^2)^2-4([K^2+v_L^2][K^2+v_R^2]-4(k_1k_1'+k_2k_2')^2)}\right) \tag{3.14}$$

therefore, we can obtain the exact eigenvalues of $\mathcal{M}_{CB}$:

$$\begin{aligned}
\mathcal{M}^2_{W_1} &= \frac{g^2}{4}\lambda_1 = \frac{g^2}{4}\left(k_1^2+k_1'^2+k_2^2+k_2'^2+\frac{v_L^2+v_R^2}{2}-\sqrt{\Delta}\right) \\
\mathcal{M}^2_{W_2} &= \frac{g^2}{4}\lambda_2 = \frac{g^2}{4}\left(k_1^2+k_1'^2+k_2^2+k_2'^2+\frac{v_L^2+v_R^2}{2}+\sqrt{\Delta}\right),
\end{aligned} \tag{3.15}$$

where we have define: $\Delta \equiv 4(k_1k_1'+k_2k_2')^2+\left(\frac{v_R^2-v_L^2}{2}\right)^2$.

the eigenvectors of $\mathcal{M}^2_{CB}$ (physical states):

$$W^+_{1\mu} \;=\; \left(\frac{v_R^2-v_L^2+2\sqrt{\Delta}}{2N}\right)W^+_{\mu L}+\left(\frac{2B}{N}\right)W^+_{\mu R} \tag{3.16}$$

$$W^+_{2\mu} \;=\; = \left(\frac{-2B}{N}\right)W^+_{\mu L}+\left(\frac{v_R^2-v_L^2+2\sqrt{\Delta}}{2N}\right)W^+_{\mu R}, \tag{3.17}$$

we can express these relations as:

$$\begin{aligned}
W^+_{1\mu} &= W^+_{\mu L}\cos\xi + W^+_{\mu R}\sin\xi, \\
W^+_{2\mu} &= -W^+_{\mu L}\sin\xi + W^+_{\mu R}\cos\xi,
\end{aligned} \tag{3.18}$$

where:

$$\begin{aligned}
\sin\xi &= \frac{2B}{N}, \\
\cos\xi &= \frac{v_R^2-v_L^2+2\sqrt{\Delta}}{2N}, \\
B &\equiv k_1k_1'+k_2k_2', \\
N &\equiv \sqrt{4\,B^2+\left(\frac{v_R^2-v_L^2+2\sqrt{\Delta}}{2}\right)^2},
\end{aligned}$$

$\xi$: represents the mixing angle between the bosons $W_R$ and $W_L$. We also can express the symmetry states $(W^+_{\mu L},\ W^+_{\mu R})$ as a function of physical states $(W^+_{1\mu},\ W^+_{2\mu})$, that is:

$$\begin{aligned}
W^+_{\mu L} &= W^+_{1\mu}\cos\xi - W^+_{2\mu}\sin\xi, \\
W^+_{\mu R} &= W^+_{1\mu}\sin\xi + W^+_{2\mu}\cos\xi,
\end{aligned} \tag{3.19}$$

**Neutral Bosons mass matrix:**

$$\mathcal{M}^2_{NB} = \begin{pmatrix} \frac{g^2}{4}(K^2+v_L^2) & -\frac{g^2}{4}K^2 & -\frac{g'\,g}{4}v_L^2 \\ -\frac{g^2}{4}K^2 & \frac{g^2}{4}(K^2+v_R^2) & -\frac{g'g}{4}v_R^2 \\ -\frac{g'\,g}{4}v_L^2 & -\frac{g'\,g}{4}v_R^2 & \frac{g'^2}{4}(v_L^2+v_R^2) \end{pmatrix},$$

remember that $K^2 \equiv k_1^2 + k_1'^2 + k_2^2 + k_2'^2$.

In the same way as the previous item, the diagonalization of the matrix:

$$\det|\mathcal{M}^2_{NB} - \lambda I \ \ | = 0 \qquad \text{(charateristic equation)},$$

solving the characteristic equation we obtain the exact eigenvalues (masses of neutral gauge bosons):

$$\begin{aligned} M^2_{Z_1} &= \lambda_1 = \frac{1}{4}\left[g^2\,K^2 + \frac{1}{2}\left(g^2+g'^2\right)\left(v_L^2+v_R^2\right) - \frac{1}{2}\sqrt{\Delta'}\right] \\ M^2_{Z_2} &= \lambda_2 = \frac{1}{4}\left[g^2\,K^2 + \frac{1}{2}\left(g^2+g'^2\right)\left(v_L^2+v_R^2\right) + \frac{1}{2}\sqrt{\Delta'}\right] \\ M^2_{A_\mu} &= \lambda_3 = 0. \end{aligned} \tag{3.20}$$

where we have defined:

$$\Delta' \equiv \left(g^2+g'^2\right)^2\left(v_L^2+v_R^2\right)^2 - 4g^2v_L^2v_R^2\left(g^2+2g'^2\right) - 4\,K^2g^2g'^2\left(v_L^2+v_R^2\right) + 4\,g^4\,K^4,$$

we can observe that $\lambda_3 = 0$ corresponds to that of the Photon.

Obtaining the eigenvectors (physical states: $A_\mu$, $Z_{1\mu}$, $Z_{2\mu}$) of $\mathcal{M}_{NB}$ as a function of symmetry states ($W^{3L}_\mu$, $W^{3R}_\mu$, $B_\mu$):

$$\begin{aligned} A_\mu &= \frac{1}{N_1'}[a_{11}W^{3L}_\mu + a_{12}W^{3R}_\mu + a_{13}\,B_\mu], \\ Z_{1\mu} &= \frac{1}{N_2'}[a_{21}W^{3L}_\mu + a_{22}W^{3R}_\mu + a_{23}\,B_\mu], \\ Z_{2\mu} &= \frac{1}{N_3'}[a_{31}W^{3L}_\mu + a_{32}W^{3R}_\mu + a_{33}\,B_\mu], \end{aligned} \tag{3.21}$$

where:

$$\begin{aligned} a_{11} &= 1, \\ a_{12} &= 1, \\ a_{13} &= g/g', \\ a_{21} &= g\,g'v_R^2, \\ a_{22} &= g\,g'v_R^2\,D_1, \\ a_{23} &= -K^2g^2 + 4C_1\,D_1, \\ a_{31} &= g\,g'v_R^2 \\ a_{32} &= g\,g'v_R^2\,D_2, \\ a_{33} &= -K^2g^2 + 4C_2\,D_2, \end{aligned} \tag{3.22}$$

also, we have defined:

$$
\begin{aligned}
N'_1 &= \sqrt{a_{11}^2 + a_{12}^2 + a_{13}^2} = \sqrt{2 + (g/g')^2}, \\
N'_2 &= \sqrt{a_{21}^2 + a_{22}^2 + a_{23}^2} = \sqrt{(g\, g' v_R^2)^2 + (g\, g' v_R^2\, D_1)^2 + (-K^2 g^2 + 4C_1\, D_1)^2}, \\
N'_3 &= \sqrt{a_{31}^2 + a_{32}^2 + a_{33}^2} = \sqrt{(g\, g' v_R^2)^2 + (g\, g' v_R^2\, D_2)^2 + (-K^2 g^2 + 4C_2\, D_2)^2}, \\
C_1 &\equiv -\frac{1}{8}(g'^2 + g^2)v_L^2 - \frac{1}{8}(g'^2 - g^2)v_R^2 + \frac{1}{8}\sqrt{\Delta'}, \\
C_2 &\equiv -\frac{1}{8}(g'^2 + g^2)v_L^2 - \frac{1}{8}(g'^2 - g^2)v_R^2 - \frac{1}{8}\sqrt{\Delta'}, \\
, D_1 &\equiv \frac{v_L^2 v_R^2 (g^2 - g'^2) - v_R^4 (g'^2 + g^2) + 2g^2 K^2\, v_L^2 + v_R^2 \sqrt{\Delta'}}{2g^2 K^2\, v_R^2 - v_L^4 (g'^2 + g^2) - (g'^2 - g^2) v_R^2 v_L^2 + v_L^2 \sqrt{\Delta'}}, \\
D_2 &\equiv \frac{v_L^2 v_R^2 (g^2 - g'^2) - v_R^4 (g'^2 + g^2) + 2g^2 K^2\, v_L^2 - v_R^2 \sqrt{\Delta'}}{2g^2 K^2\, v_R^2 - v_L^4 (g'^2 + g^2) - (g'^2 - g^2) v_R^2 v_L^2 - v_L^2 \sqrt{\Delta'}},
\end{aligned}
$$

we can express the equations 3.21 in a matrix form:

$$
\begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix} = \underbrace{\begin{pmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{pmatrix}}_{=P} \begin{pmatrix} W_\mu^{3L} \\ W_\mu^{3R} \\ B_\mu \end{pmatrix} \tag{3.23}
$$

where $P$ represents a ortogonal matrix:

$$
P = \begin{pmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{pmatrix} \tag{3.24}
$$

to note that $a'_{ij} = \frac{a_{ij}}{N'_i}$. Moreover, we need the inverse matrix $P^{-1}$:

$$
\begin{pmatrix} W_\mu^{3L} \\ W_\mu^{3R} \\ B_\mu \end{pmatrix} = P^{-1} \begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix} \tag{3.25}
$$

the matrix $P$ explicitly:

$$
P = \begin{pmatrix} 1/N'_1 & 1/N'_1 & (g/g')/N'_1 \\ g\, g' v_R^2 / N'_2 & g\, g' v_R^2\, D_1/N'_2 & (-K^2 g^2 + 4C_1\, D_1)/N'_2 \\ g\, g' v_R^2 / N'_3 & g\, g' v_R^2\, D_2/N'_3 & (-K^2 g^2 + 4C_2\, D_2)/N'_3 \end{pmatrix}
$$

but $P^{-1} = P^T$, because the matrix $P$ is an ortogonal matrix.

$$
P^{-1} = P^T = \begin{pmatrix} 1/N'_1 & g\, g' v_R^2 / N'_2 & g\, g' v_R^2 / N'_3 \\ 1/N'_1 & g\, g' v_R^2\, D_1/N'_2 & g\, g' v_R^2\, D_2/N'_3 \\ (g/g')/N'_1 & (-K^2 g^2 + 4C_1\, D_1)/N'_2 & (-K^2 g^2 + 4C_2\, D_2)/N'_3 \end{pmatrix},
$$

then:

$$
\begin{pmatrix} W_\mu^{3L} \\ W_\mu^{3R} \\ B_\mu \end{pmatrix} = \begin{pmatrix} 1/N'_1 & g\, g' v_R^2 / N'_2 & g\, g' v_R^2 / N'_3 \\ 1/N'_1 & g\, g' v_R^2\, D_1/N'_2 & g\, g' v_R^2\, D_2/N'_3 \\ (\frac{g}{g'})/N'_1 & (-K^2 g^2 + 4C_1\, D_1)/N'_2 & (-K^2 g^2 + 4C_2\, D_2)/N'_3 \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix}
$$

then, we have:

$$\begin{aligned}
W_\mu^{3L} &= (1/N_1')\,A_\mu + \left(g\,g' v_R^2/N_2'\right) Z_{1\mu} + \left(g\,g' v_R^2/N_3'\right) Z_{2\mu} \\
W_\mu^{3R} &= (1/N_1')\,A_\mu + \left(g\,g' v_R^2\,D_1/N_2'\right) Z_{1\mu} + \left(g\,g' v_R^2\,D_2/N_3'\right) Z_{2\mu}, \\
B_\mu &= \left(\frac{g}{N_1' g'}\right) A_\mu + \left([-K^2 g^2 + 4C_1\,D_1]/N_2'\right) Z_{1\mu} + \left([-K^2 g^2 + 4C_2\,D_2]/N_3'\right) Z_{2\mu},
\end{aligned} \tag{3.26}$$

However, it is important to express the symmetry states as a function of the parameters $v_R$, $v_R$, $g$, and $g'$, therefore, the transformation matrix can be written:

$$\begin{pmatrix} W_\mu^{3L} \\ W_\mu^{3R} \\ B_\mu \end{pmatrix} = \begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix}$$

That is, we have (exactly):

$$\begin{aligned}
W_\mu^{3L} &= n_{11} A_\mu + n_{12} Z_{1\mu} + n_{13} Z_{2\mu} \\
W_\mu^{3R} &= n_{21} A_\mu + n_{22} Z_{1\mu} + n_{23} Z_{2\mu}, \\
B_\mu &= n_{31} A_\mu + n_{32} Z_{1\mu} + n_{33} Z_{2\mu},
\end{aligned} \tag{3.27}$$

where:

$$\begin{aligned}
n_{11} &= \frac{1}{\sqrt{2 + \frac{g^2}{g'^2}}} \\
n_{12} &= 1/\Big(\Big[2\,g^2 K^4 - (g^2 + g'^2) K^2 v_L^2 + (g^2 + g'^2) v_R^4 + v_R^2 \left(-\sqrt{\Gamma} - v_L^2 (g^2 + g'^2)\right. \\
&+ \left.K^2 (g^2 - g'^2)\right) - K^2 \sqrt{\Gamma}\Big]^2 / \left(-2 g^2 K^4 + 2 g'^2 v_L^2 v_R^2 + (g'^2 - g^2)(v_L^2 + v_R^2) K^2 + K^2 \sqrt{\Gamma}\right)^2 \\
&+ \frac{g'^2 (2 g^2 K^2 - (g^2 - g'^2) v_L^2 - (g^2 + g'^2) v_R^2 + \sqrt{\Gamma})^2}{4 g^2 (g'^2 v_L^2 - g^2 K^2)^2} + 1\Big)^{1/2} \\
n_{13} &= 1/\Big(\Big[2\,g^2 K^4 - (g^2 + g'^2) K^2 v_L^2 + (g^2 + g'^2) v_R^4 + v_R^2 \left(\sqrt{\Gamma} - v_L^2 (g^2 + g'^2)\right. \\
&+ \left.K^2 (g^2 - g'^2)\right) + K^2 \sqrt{\Gamma}\Big]^2 / \left(-2 g^2 K^4 + 2 g'^2 v_L^2 v_R^2 + (g'^2 - g^2)(v_L^2 + v_R^2) K^2 - K^2 \sqrt{\Gamma}\right)^2 \\
&+ \frac{g'^2 (2 g^2 K^2 - (g^2 - g'^2) v_L^2 - (g^2 + g'^2) v_R^2 - \sqrt{\Gamma})^2}{4 g^2 (g'^2 v_L^2 - g^2 K^2)^2} + 1\Big)^{1/2} \\
n_{21} &= \frac{1}{\sqrt{2 + \frac{g^2}{g'^2}}} \\
n_{22} &= \Big[2\,g^2 K^4 - (g^2 + g'^2) K^2 v_L^2 + (g^2 + g'^2) v_R^4 + v_R^2 \left(-\sqrt{\Gamma} - v_L^2 (g^2 + g'^2) + K^2 (g^2 - g'^2)\right) - K^2 \sqrt{\Gamma}\Big] / \\
&\Big\{\Big[K^2 \sqrt{\Gamma} + K^2 (g'^2 - g^2)(v_L^2 + v_R^2) + 2 g'^2 v_R^2 v_L^2 - 2 g^2 K^4\Big] \Big(\big(2 g^2 K^4 + (g^2 + g'^2)(v_R^4 - K^2 v_L^2) \\
&+ v_R^2 [-\sqrt{\Gamma} - (g^2 + g'^2) v_L^2 + (g^2 - g'^2) K^2] - K^2 \sqrt{\Gamma}\big)^2 / [-2 g^2 K^4 + 2 g'^2 v_L^2 v_R^2 + (g'^2 - g^2) \times \\
&K^2 (v_L^2 + v_R^2) + K^2 \sqrt{\Gamma}]^2 + \frac{g'^2 (2 g^2 K^2 - (g^2 - g'^2) v_L^2 - g^2 + g'^2) v_R^2 + \sqrt{\Gamma})^2}{4 g^2 (g'^2 v_L^2 - g^2 K^2)^2} + 1\Big)^{1/2}\Big\}
\end{aligned}$$

$$
\begin{aligned}
n_{23} &= \Big[2\,g^2K^4-(g^2+g'^2)K^2v_L^2+(g^2+g'^2)v_R^4+v_R^2\Big(\sqrt{\Gamma}-v_L^2(g^2+g'^2)+K^2(g^2-g'^2)\Big)+K^2\sqrt{\Gamma}\Big]\,/\\
&\quad\Big\{\Big[-K^2\sqrt{\Gamma}+K^2(g'^2-g^2)(v_L^2+v_R^2)+2g'^2v_R^2v_L^2-2g^2K^4\Big]\Big(\big(2g^2K^4+(g^2+g'^2)(v_R^4-K^2v_L^2)\\
&\quad+v_R^2[\sqrt{\Gamma}-(g^2+g'^2)v_L^2+(g^2-g'^2)K^2]+K^2\sqrt{\Gamma}\big)^2/[-2g^2K^4+2g'^2v_L^2v_R^2+(g'^2-g^2)\times\\
&\quad K^2(v_L^2+v_R^2)-K^2\sqrt{\Gamma}]^2+\frac{g'^2(2g^2K^2-(g^2-g'^2)v_L^2-g^2+g'^2)v_R^2-\sqrt{\Gamma})^2}{4g^2(g'^2v_L^2-g^2K^2)^2}+1\Big)^{1/2}\Big\}\\
n_{31} &= \frac{g}{g'\,\sqrt{2+\frac{g^2}{g'^2}}}\\
n_{32} &= g'\Big(2g^2K^2-(g'^2-g^2)v_L^2-(g^2+g'^2)v_R^2+\sqrt{\Gamma}\Big)\,/\,\big\{2g(g'^2v_L^2-g^2K^2)\,\big[(2g^2K^4+(g^2+g'^2)\times\\
&\quad(v_R^4-K^2v_L^2)+v_R^2(-\sqrt{\Gamma}-(g^2+g'^2)v_L^2+(g^2-g'^2)K^2)-K^2\sqrt{\Gamma})^2/(-2g^2K^4+2g'^2v_L^2v_R^2+\\
&+\quad(g'^2-g^2)K^2(v_R^2+v_L^2)+K^2\sqrt{\Gamma})^2+\frac{g'^2(2g^2K^2-(g'^2-g^2)v_L^2-(g^2+g'^2)v_R^2+\sqrt{\Gamma})}{4g^2(g'^2v_L^2-g^2K^2)^2}+1\Big]^{1/2}\Big\}\\
n_{33} &= g'\Big(2g^2K^2-(g'^2-g^2)v_L^2-(g^2+g'^2)v_R^2-\sqrt{\Gamma}\Big)\,/\,\big\{2g(g'^2v_L^2-g^2K^2)\,\big[(2g^2K^4+(g^2+g'^2)\times\\
&\quad(v_R^4-K^2v_L^2)+v_R^2(\sqrt{\Gamma}-(g^2+g'^2)v_L^2+(g^2-g'^2)K^2)+K^2\sqrt{\Gamma})^2/(-2g^2K^4+2g'^2v_L^2v_R^2+\\
&+\quad(g'^2-g^2)K^2(v_R^2+v_L^2)-K^2\sqrt{\Gamma})^2+\frac{g'^2(2g^2K^2-(g'^2-g^2)v_L^2-(g^2+g'^2)v_R^2-\sqrt{\Gamma})}{4g^2(g'^2v_L^2-g^2K^2)^2}+1\Big]^{1/2}\Big\}
\end{aligned}
$$

where:

$$
\Gamma = (g^2+g'^2)^2(v_L^2+v_R^2)^2-4g^2g'^2K^2(v_L^2+v_R^2)-4g^2(g^2+2g'^2)v_L^2v_R^2+4g^4K^4
$$

Remember that: $K^2\equiv k_1^2+k_2^2+k_1'^2+k_2'^2$.

### 3.2.2 Approximate analysis of the masses and physical states of the gauge bosons

The previous results are given in an exact way, so here we will consider the analysis of the masses of the gauge bosons and the eigenstates in an approximate way, that is, when we consider large values of $v_R$ compared with the other VEVs.

Regarding that $v_R$ is larger than others VEVs, $v_R^2\gg v_L^2$, $A'$, from the expression (3.27) we have:

$$
\begin{aligned}
W_\mu^{3L} &= a_1\,A_\mu+a_2\,Z_{1\mu}+a_3\,Z_{2\mu}\\
W_\mu^{3R} &= b_1\,A_\mu+b_2\,Z_{1\mu}+b_3\,Z_{2\mu}\\
B_\mu &= c_1\,A_\mu+c_2\,Z_{1\mu}+c_3\,Z_{2\mu}
\end{aligned}
\tag{3.28}
$$

where:

$$
\begin{aligned}
a_1 &= \frac{g'}{\sqrt{g^2+2g'^2}} \\
a_2 &\approx \sqrt{\frac{g^2+g'^2}{g^2+2\,g'^2}} - \frac{g^2\,\sqrt{g^2+2g'^2}\,\sqrt{g^2+g'^2}(g^2\,K^2-g'^2v_L^2)^2}{2\,\left(g^2+g'^2\right)^4\,v_R^4} \\
a_3 &\approx -\frac{g\,(g^2\,K^2-v_L^2\,g'^2)}{v_R^2\,(g^2+g'^2)^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
b_1 &= \frac{g'}{\sqrt{g^2+2g'^2}} \\
b_2 &\approx \frac{-g'^2}{\sqrt{g'^2+g^2}\sqrt{2g'^2+g^2}} + g^2\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}}\left(\frac{K^2g^2-v_L^2g'^2}{v_R^2(g^2+g'^2)^2}\right) \\
b_3 &\approx \frac{g}{\sqrt{g^2+g'^2}} + \frac{gg'^2\sqrt{g^2+g'^2}[K^2g^2-g'^2v_L^2]}{v_R^2(g^2+g'^2)^3}
\end{aligned}
\tag{3.29}
$$

$$
\begin{aligned}
c_1 &= \frac{g}{g'\sqrt{2+\frac{g^2}{g'^2}}} \\
c_2 &\approx -\frac{g\,g'}{\sqrt{g^2+g'^2}\sqrt{g^2+2g'^2}} - gg'\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}}\,\frac{K^2g^2-g'^2v_L^2}{v_R^2(g^2+g'^2)^2} \\
c_3 &\approx -\frac{g'}{\sqrt{g^2+g'^2}} + g'g^2\,\frac{K^2g^2-v_L^2g'^2}{v_R^2(g^2+g'^2)^{5/2}}
\end{aligned}
$$

Remember we have defined from de the expresion (2.7): $r \equiv g'/g$. The electric charge operator is:

$$
e = \frac{g\,g'}{\sqrt{g^2+2g'^2}} = \frac{g'}{\sqrt{1+2r^2}}, \tag{3.30}
$$

and the angle $\theta$, expression (2.6), we can write:

$$
\sin\theta = \frac{g'}{\sqrt{g^2+2g'^2}} = \frac{r}{\sqrt{1+2r^2}}, \qquad \cos\theta = \sqrt{\frac{g^2+g'^2}{g^2+2g'^2}} = \sqrt{\frac{1+r^2}{1+2r^2}}, \tag{3.31}
$$

hence, from the previous equations we obtain:

$$
r^2 = r_\theta^2 = s_\theta^2/c_{2\theta} = \frac{s_\theta^2}{1-2s_\theta^2}
$$

This last expression was commented in the 2.2.1 section, the matching condition, $s_\theta^2 < 1/2$.

Notice that:

$$
\tan\theta \equiv t_\theta = \frac{r}{\sqrt{1+r^2}}. \tag{3.32}
$$

It is posible to express the neutral gauge bosones masses, given by the expression (3.20) as a function of $r$, like that:

$$\begin{aligned} M_{Z_1}^2 &= \frac{g^2}{4}\left[K^2+\frac{1}{2}\left(1+r^2\right)\left(v_L^2+v_R^2\right)-\frac{1}{2}\sqrt{\Delta}\right] \\ M_{Z_2}^2 &= \frac{g^2}{4}\left[K^2+\frac{1}{2}\left(1+r^2\right)\left(v_L^2+v_R^2\right)+\frac{1}{2}\sqrt{\Delta}\right] \\ M_{A_\mu}^2 &= 0. \end{aligned} \tag{3.33}$$

remmember that:

$$\Delta \equiv \left(1+r^2\right)^2\left(v_L^2+v_R^2\right)^2-4\,v_L^2v_R^2\left(1+2r^2\right)-4\,K^2r^2\left(v_L^2+v_R^2\right)+4\,K^4,$$

As it is known, the symmetry eigenstates are linear combinations of the mass eigenstates as follows:

$$\begin{pmatrix} W_\mu^{3L} \\ W_\mu^{3R} \\ B_\mu \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z_1^\mu \\ Z_2^\mu \end{pmatrix}, \tag{3.34}$$

the coefficients, $a_i$, $b_i$ and $c_i$ (which are approximate values), are given in (3.29), and we can express them as a function of $\theta$:

$$\begin{aligned} a_1 &= s_\theta, \quad a_2 \approx c_\theta\left[1-\frac{c_{2\theta}^3}{2v_R^4c_\theta^6}\left(K^2-\frac{s_\theta^2}{c_{2\theta}}v_L^2\right)^2\right], \quad a_3 \approx -\left(\frac{c_{2\theta}}{c_\theta^2}\right)^{3/2}\left[K^2-\frac{s_\theta^2}{c_{2\theta}}v_L^2\right]\frac{1}{v_R^2}, \\ b_1 &= s_\theta, \quad b_2 \approx -s_\theta\,t_\theta+\frac{c_{2\theta}^2}{v_R^2c_\theta^5}\left(K^2-\frac{v_L^2s_\theta^2}{c_{2\theta}}\right), \quad b_3 \approx \frac{\sqrt{c_{2\theta}}}{c_\theta}\left\{1+\frac{c_{2\theta}\,s_\theta^2}{v_R^2\,c_\theta^4}\left[K^2-v_L^2\,\frac{s_\theta^2}{c_{2\theta}}\right]\right\}, \\ c_1 &= \sqrt{c_{2\theta}}, \quad c_2 \approx -t_\theta\sqrt{c_{2\theta}}\left[1+\frac{c_{2\theta}}{v_R^2\,c_\theta^4}\left(K^2-\frac{s_\theta^2\,v_L^2}{c_{2\theta}}\right)\right], \quad c_3 \approx -t_\theta\left[1-\frac{c_{2\theta}^2}{v_R^2\,c_\theta^4}\left(K^2-\frac{s_\theta^2}{c_{2\theta}}v_L^2\right)\right], \end{aligned} \tag{3.35}$$

where: $t_\theta \equiv \tan\theta$, $s_\theta \equiv \sin\theta$, $c_\theta \equiv \cos\theta$, $c_{2\theta} \equiv \cos 2\theta$, $t_{2\theta} \equiv \tan 2\theta$.

We can also express the previous matrix elements in other way, that is, making the following definition:

$$\begin{aligned} x &= \frac{K^2}{v_R^2}, \\ y &= \frac{v_L^2}{v_R^2}, \\ z &= \frac{\bar{K}_L^2}{v_R^2}, \\ \phi &= \frac{c_{2\theta}^{3/2}}{c_\theta^3}\left(x-\frac{s_\theta^2}{c_{2\theta}}y\right), \end{aligned} \tag{3.36}$$

the matrix 3.34, can be expressed:

$$\begin{pmatrix} W_\mu^{3L} \\ W_\mu^{3R} \\ B_\mu \end{pmatrix} = \begin{pmatrix} n & n_{12} & n_{13} \\ n & n_{22} & n_{23} \\ n' & n_{32} & n_{33} \end{pmatrix}\begin{pmatrix} A^\mu \\ Z_1^\mu \\ Z_2^\mu \end{pmatrix}, \tag{3.37}$$

where:

$$
\begin{aligned}
n &= s_\theta, \quad n' = \sqrt{c_{2\theta}}, \quad n_{12} \approx c_\theta, \quad n_{13} \approx \phi, \\
n_{22} &\approx -s_\theta\, t_\theta \left[1 - \phi \frac{\sqrt{c_{2\theta}}}{s_\theta^2\, c_\theta}\right], \quad n_{23} \approx \frac{\sqrt{c_{2\theta}}}{c_\theta}\left[1 + \phi \frac{c_\theta\, t_\theta^2}{\sqrt{c_{2\theta}}}\right] \\
n_{32} &\approx -t_\theta \sqrt{c_{2\theta}} \left[1 + \frac{\phi}{c_\theta \sqrt{c_{2\theta}}}\right], \quad n_{33} \approx -t_\theta \left[1 - \phi \frac{\sqrt{c_{2\theta}}}{c_\theta}\right]
\end{aligned} \tag{3.38}
$$

In the case of the charge boson masses, using the Wx-maxima software, both masses given in (3.15) can be expanded in Taylor series regarding large values of $v_R$:

$$
\begin{aligned}
\mathcal{M}_{W_1}^2 &= \frac{g^2}{4}\left(K^2 + \frac{v_L^2 + v_R^2}{2} - \sqrt{\Delta}\right) \approx \frac{g^2}{4}\left(K^2 + \frac{v_L^2}{2}\right) \\
\mathcal{M}_{W_2}^2 &= \frac{g^2}{4}\left(K^2 + \frac{v_L^2 + v_R^2}{2} + \sqrt{\Delta}\right) \approx \frac{g^2}{4} v_R^2,
\end{aligned} \tag{3.39}
$$

where: $\Delta \equiv 4(k_1 k_1' + k_2 k_2')^2 + \left(\frac{v_R^2 - v_L^2}{2}\right)^2$. Notice that if all VEVs are positive, then $M_{W_2}^2 \gg M_{W_1}^2$, and we identify the $W_1^\pm$ of the left-right model as the $W^\pm$ of the SM.

### 3.2.3 Calculation of the minimum value of $v_R$

Regarding the exact masses of the $M_{W_1}$ and $M_{Z_1}$ vectorial bossons:

$$
\frac{M_{W_1}}{M_{Z_1}} = \sqrt{\frac{x + \frac{1}{2}(1+y) - \sqrt{\Delta}}{x + \frac{1}{2}(1+y)(1+r^2) - \frac{1}{2}\sqrt{\Omega}}}, \tag{3.40}
$$

where:

$$
\Delta = 4z^2 + \frac{1}{4}(y-1)^2, \tag{3.41}
$$

$$
\Omega = (1+y)^2(1+r^2)^2 - 4y(1+2r^2) - 4xr^2(1+y) + 4x^2, \tag{3.42}
$$

remember that:

$$
\begin{aligned}
x &= \frac{K^2}{v_R^2}, \\
y &= \frac{v_L^2}{v_R^2}, \\
z &= \frac{\bar{K}^2}{v_R^2},
\end{aligned} \tag{3.43}
$$

where: $\bar{K}^2 = k_1 k_1' + k_2 k_2'$, and $K^2 = k_1^2 + k_1'^2 + k_2^2 + k_2'^2$

**Relating the parameters $x$ and $z$:**

Regarding: $k_1 \approx k_1' \approx 0$ and $k_2 \approx k_2'$, we have:

$$
\bar{K}^2 \approx k_2^2, \tag{3.44}
$$

$$
K^2 \approx 2\, k_2^2, \tag{3.45}
$$

In addition:

$$K^2 \approx 2\,\bar{K}^2. \tag{3.46}$$

then, from (3.43) we obtain the following relations:

$$x = 2\,z \quad \rightarrow \quad z = \frac{x}{2}. \tag{3.47}$$

the expression (3.41) take the form:

$$\Delta = x^2 + \frac{1}{4}(y-1)^2, \tag{3.48}$$

**Looking for a minimum value for $v_R$:**

Making $v_L = 0$, it is obtained $y = 0$. Then, the equation (3.40) take the form:

$$\frac{M_{W_1}}{M_{Z_1}} = \sqrt{\frac{x + \frac{1}{2} - \sqrt{\Delta}}{x + \frac{1}{2}(1+r^2) - \frac{1}{2}\sqrt{\Omega}}} = 0.88147 \pm 0.00013,\,^{1} \tag{3.49}$$

where:

$$\Delta = x^2 + \frac{1}{4}, \tag{3.50}$$

$$\Omega = (1+r^2)^2 - 4xr^2 + 4x^2. \tag{3.51}$$

According to the Standard Model, we can regard the following values:

$$r = \frac{g'}{g} = 0.6314, \tag{3.52}$$

$$k_2 = \frac{v_{SM}}{\sqrt{2}} = \frac{246\,\text{GeV}}{\sqrt{2}}, \tag{3.53}$$

$$K = \sqrt{2}\,k_2 = 246\,\text{GeV}. \tag{3.54}$$

We know: $x = \frac{K^2}{v_R^2}$, then:

$$v_R = \frac{K}{\sqrt{x}} = \frac{246}{\sqrt{x}}\text{GeV}$$

1. For the case: $\frac{M_{W_1}}{M_{Z_1}} = 0.88147 \pm 0.00026^2$, $(2\,\sigma)$ we obtain:
   Where the interception of the two curves, we get the value: $x = 0.0001016$.

   With this value we obtain the minimum expectation value $v_R$, that is:

   $$v_R = 24.00\ \text{TeV}$$

2. For the case: $\frac{M_{W_1}}{M_{Z_1}} = 0.88147 \pm 0.00039$, $(3\,\sigma)$ we obtain:
   In the same way as the previous item, the interception of the two curves gives the following value of $x = 0.0007058$.

   With this value we obtain the minimum expectation value $v_R$, that is:

   $$v_R = 9.26\ \text{TeV}$$

[2] values taken from the PDG-2020

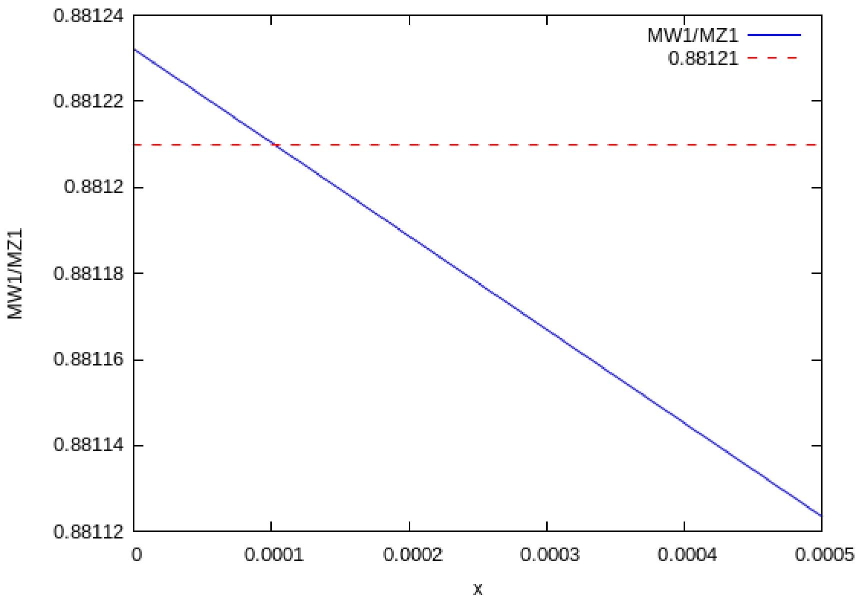


Figure 3.1: Estimation of x, using $2\sigma$ approximation in $\frac{M_{W_1}}{M_{Z_1}}$

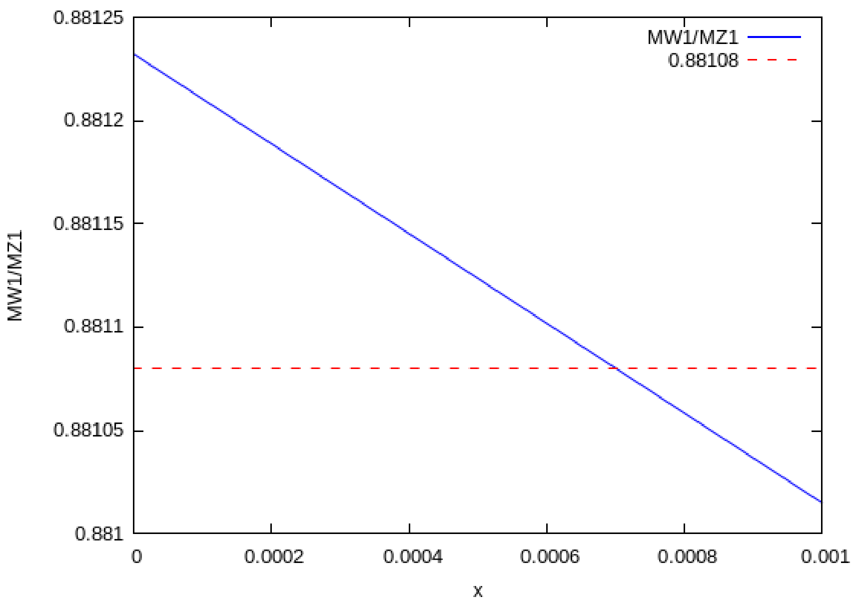


Figure 3.2: Estimation of x, using $3\sigma$ approximation in $\frac{M_{W_1}}{M_{Z_1}}$

We can gather the results in a single graph:

In summary, we can consider the following condition (lower limit) to $v_R \geq 24.00$ TeV for getting the $W_2$ and $Z_2$ masses.

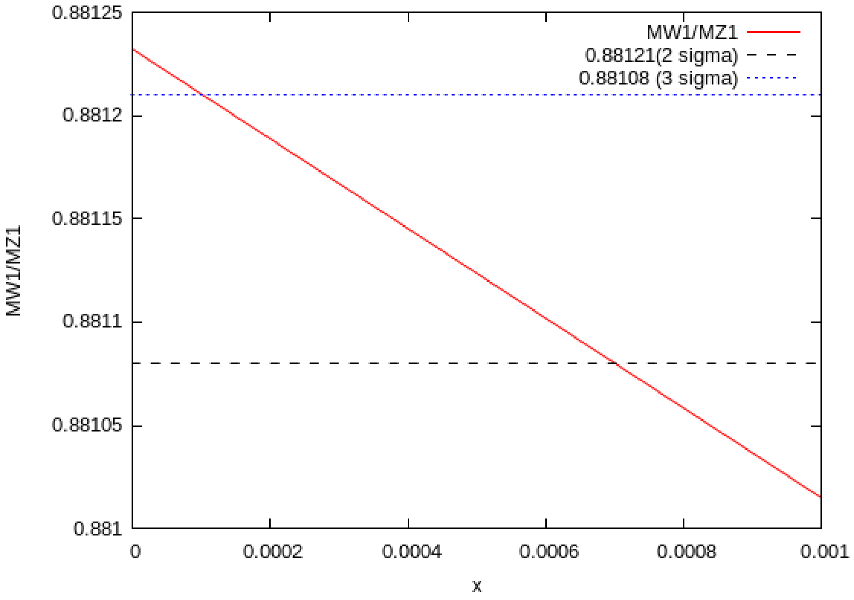


Figure 3.3: Estimation of x gathering the two previous graphics

### 3.2.4 Calculation of the masses of the $W_2$ and $Z_2$ bosons

Knowing the general expression of the bosons masses:

$$M^2_{W_2} = \frac{g^2\, v_R^2}{4}\left(x + \frac{1+y}{2} + \sqrt{\Delta}\right), \tag{3.55}$$

$$M^2_{Z_2} = \frac{g^2\, v_R^2}{4}\left(x + \frac{1}{2}(1+y)(1+r^2) + \frac{1}{2}\sqrt{\Omega}\right), \tag{3.56}$$

where $\Omega$ and $\Delta$ are given by the expressions (3.41) and (3.43).

Given the values: $v_L = 0$, then $y = 0$, we get the following expressions:

$$M^2_{W_2} = \frac{g^2\, v_R^2}{4}\left(x + \frac{1}{2} + \sqrt{\Delta}\right), \tag{3.57}$$

$$M^2_{Z_2} = \frac{g^2\, v_R^2}{4}\left(x + \frac{1}{2}(1+r^2) + \frac{1}{2}\sqrt{\Omega}\right), \tag{3.58}$$

According to the SM:

$$G_F = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2} = 1.16637 \times 10^{-5}\ \ \mathrm{GeV}^{-2}, \tag{3.59}$$

then:

$$g^2 = 4\sqrt{2} G_F M_W^2 \tag{3.60}$$

where: $M_W = 80.379$ GeV, then:

$$g^2 = 0.4263 \tag{3.61}$$

Also, using the minimum value of $v_R = 24.00$ TeV, we obtain:

$$\begin{aligned} M_{W_2} &= 7.835\,\mathrm{TeV}. \\ M_{Z_2} &= 9.284\,\mathrm{TeV} \end{aligned} \tag{3.62}$$

Notice that only in the limit $v_R \to \infty$ does the angle $\theta$ in this model have a relation with the $\theta_W$ of the SM. However, it is important that $v_R$ is kept to be large but finite in order to obtain a lower bound on the right-handed vector bosons, $W_2$ and $Z_2$ [37] and the respective coupling with fermions. If $\chi_L$ is an inert doublet, we simply put $v_L = 0$, or, equivalently, $y = 0$, in the above expressions.

### 3.2.5 Calculation of the angle $\xi$ for the charged bosons

Recall that:

$$\sin(\xi) = \frac{2B}{N} = \frac{2(k_1 k_1' + k_2 k_2')}{\sqrt{4B^2 + \left(\frac{v_R^2 - v_L^2 + 2\sqrt{\Delta}}{2}\right)^2}}, \quad (3.63)$$

$$\cos(\xi) = \frac{v_R^2 - v_L^2 + 2\sqrt{\Delta}}{2N} = \frac{v_R^2 - v_L^2 + 2\sqrt{\Delta}}{2\sqrt{4B^2 + \left(\frac{v_R^2 - v_L^2 + 2\sqrt{\Delta}}{2}\right)^2}} \quad (3.64)$$

Considering the same aproximation of the previous section, we may express the above equation as a function of $x$, $z$, e $y$, that is:

$$\begin{aligned} \sin(\xi) &= \frac{4z}{\sqrt{16\,z^2 + Y^2}}, \\ \cos(\xi) &= \frac{Y}{\sqrt{16\,z^2 + Y^2}}, \end{aligned} \quad (3.65)$$

where: $Y = 1 - y + 2\sqrt{\Delta}$, making $v_L = 0$ then $y = 0$, finally we obtain: $Y = 1 + 2\sqrt{\Delta}$.

Replacing the value $x = 0.0001016$ in the equation (3.65):

$$\begin{aligned} \sin(\xi) &= 0.000101599, \\ \cos(\xi) &= 0.999999994, \end{aligned} \quad (3.66)$$

thus, the angle value $\xi$ is given by:

$$\xi = 1.01599 \times 10^{-4}\,\text{rad}. \quad (3.67)$$

The mixing angle $W_L - W_R$ defined in Eq. (3.66) has an upper limit $\sin\xi < 10^{-4}$. Recent analysis comparing the experimental limits to the theoretical calculations for the total $W_2$ resonant production and the decay $W_2 \to WZ$ implies that the exclusion in the mixing angle $\xi$ is between $10^{-4}$ and $10^{-3}$, with a maximum exclusion above $\xi \simeq 6 \times 10^{-4}$ [38].

## 3.3 Coupling the leptons with the bosons:

Knowing the lagrangian density for leptons:

$$\mathcal{L}^{lep}(x) = i\,\left\{\bar{L}_l(x)\gamma^\mu D_\mu^L L_l(x) + \bar{R}_l(x)\gamma^\mu D_\mu^R R_l(x)\right\} + h.c. \quad (3.68)$$

$$\begin{aligned}\mathcal{L}^{lep} &= i\begin{pmatrix}\overline{\nu}_L & \overline{\ell}_L\end{pmatrix}\left(\gamma^\mu\partial_\mu + i\frac{g_L}{2}\gamma^\mu\overline{\tau}\cdot\overline{W}^L_\mu - i\frac{g'}{2}\gamma^\mu B_\mu\right)\begin{pmatrix}\nu_L\\ \ell_L\end{pmatrix}\\ &+ i\begin{pmatrix}\overline{\nu}_R & \overline{\ell}_R\end{pmatrix}\left(\gamma^\mu\partial_\mu + i\frac{g_R}{2}\gamma^\mu\overline{\tau}\cdot\overline{W}^R_\mu - i\frac{g'}{2}\gamma^\mu B_\mu\right)\begin{pmatrix}\nu_R\\ \ell_R\end{pmatrix}\end{aligned} \quad (3.69)$$

however:

$$\overline{\tau}\cdot\overline{W}^L_\mu = \begin{pmatrix} W^L_{3\mu} & W^L_{1\mu} - i\,W^L_{2\mu}\\ W^L_{1\mu} + i\,W^L_{2\mu} & -W^L_{3\mu}\end{pmatrix}, \quad (3.70)$$

$$\overline{\tau}\cdot\overline{W}^R_\mu = \begin{pmatrix} W^R_{3\mu} & W^R_{1\mu} - i\,W^R_{2\mu}\\ W^R_{1\mu} + i\,W^R_{2\mu} & -W^R_{3\mu}\end{pmatrix}, \quad (3.71)$$

replacing in the expression (3.69), we have:

$$\begin{aligned}\mathcal{L}^{lep} &= i\begin{pmatrix}\overline{\nu}_L & \overline{\ell}_L\end{pmatrix}\begin{pmatrix}\partial\!\!\!/ + i\frac{g_L}{2}\gamma^\mu W^L_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu & i\frac{g_L}{2}\gamma^\mu\left(W^L_{1\mu} - i\,W^L_{2\mu}\right)\\ i\frac{g_L}{2}\gamma^\mu\left(W^L_{1\mu} + i\,W^L_{2\mu}\right) & \partial\!\!\!/ - i\frac{g_L}{2}\gamma^\mu W^L_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu\end{pmatrix}\begin{pmatrix}\nu_L\\ \ell_L\end{pmatrix}\\ &+ i\begin{pmatrix}\overline{\nu}_R & \overline{\ell}_R\end{pmatrix}\begin{pmatrix}\partial\!\!\!/ + i\frac{g_R}{2}\gamma^\mu W^R_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu & i\frac{g_R}{2}\gamma^\mu\left(W^R_{1\mu} - i\,W^R_{2\mu}\right)\\ i\frac{g_R}{2}\gamma^\mu\left(W^R_{1\mu} + i\,W^R_{2\mu}\right) & \partial\!\!\!/ - i\frac{g_R}{2}\gamma^\mu W^R_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu\end{pmatrix}\begin{pmatrix}\nu_R\\ \ell_R\end{pmatrix}\end{aligned}$$

multiplying the matrices and regarding the neutrinos contribution:

$$\begin{aligned}\mathcal{L}^{lep}_{\nu_\ell} &= i\,\overline{\nu}_L\left(\partial\!\!\!/ + i\frac{g_L}{2}\gamma^\mu W^L_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu\right)\nu_L + \overline{\nu}_R\left(\partial\!\!\!/ + i\frac{g_R}{2}\gamma^\mu W^R_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu\right)\nu_R\\ &= i\,\overline{\nu}_L\,\partial\!\!\!/\nu_L + i\,\underbrace{\overline{\nu}_L\left(i\frac{g_L}{2}\gamma^\mu W^L_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu\right)\nu_L}_{(I)} + i\,\overline{\nu}_R\,\partial\!\!\!/\nu_R +\\ &+ \underbrace{i\,\overline{\nu}_R\left(i\frac{g_R}{2}\gamma^\mu W^R_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu\right)\nu_R}_{(II)}\end{aligned}$$

### 3.3.1 Coupling with the Electromagnetic field:

The interaction with the photon arises from the projection of $W_{3L}, W_{3R}$ and $B$ over $A$.

We know from the expression :

$$\begin{aligned}W^{3L}_\mu &= \left(\frac{g'}{\sqrt{g^2+2g'^2}}\right)A_\mu + \cdots\\ W^{3R}_\mu &= \left(\frac{g'}{\sqrt{g^2+2g'^2}}\right)A_\mu + \cdots\\ B_\mu &= \left(\frac{g}{\sqrt{g^2+2g'^2}}\right)A_\mu + \cdots\end{aligned} \quad (3.72)$$

**For the case of neutrinos:**

working with $(I)$ and $(II)$:

$$(I)+(II)=\frac{i}{2}\overline{\nu}_L\gamma^\mu\left(g\underbrace{W^L_{3\mu}}_{A_\mu\frac{g'}{\sqrt{g^2+2g'^2}}}-g'\underbrace{B_\mu}_{A_\mu\frac{g}{\sqrt{g^2+2g'^2}}}\right)\nu_L+\frac{i}{2}\overline{\nu}_R\gamma^\mu\left(g\underbrace{W^R_{3\mu}}_{A_\mu\frac{g'}{\sqrt{g^2+2g'^2}}}-g'\underbrace{B_\mu}_{A_\mu\frac{g}{\sqrt{g^2+2g'^2}}}\right)\nu_R \quad (3.73)$$

Where it has been necessary to do $g_L=g_R=g$, then we have:

$$\begin{aligned}
(I)+(II) &\overset{A_\mu}{=} \frac{i}{2}\underbrace{\overline{\nu}_L}_{\overline{\nu_\ell}P_R}\gamma^\mu A_\mu\left(g\frac{g'}{\sqrt{g^2+2g'^2}}-g'\frac{g}{\sqrt{g^2+2g'^2}}\right)\underbrace{\nu_L}_{P_L\nu_\ell}+\\
&+ \frac{i}{2}\underbrace{\overline{\nu}_R}_{\overline{\nu}_\ell\,PL}\gamma^\mu A_\mu\left(g\frac{g'}{\sqrt{g^2+2g'^2}}-g'\frac{g}{\sqrt{g^2+2g'^2}}\right)\underbrace{\nu_R}_{P_R\,\nu_\ell}\\
&= \frac{i}{2}\overline{\nu_\ell}P_R\gamma^\mu A_\mu\left(g\frac{g'}{\sqrt{g^2+2g'^2}}-g'\frac{g}{\sqrt{g^2+2g'^2}}\right)P_L\nu_\ell+\\
&+ \frac{i}{2}\overline{\nu}_\ell P_L\gamma^\mu A_\mu\left(g\frac{g'}{\sqrt{g^2+2g'^2}}-g'\frac{g}{\sqrt{g^2+2g'^2}}\right)P_R\nu_\ell\\
&= \frac{i}{2}\overline{\nu_\ell}\gamma^\mu A_\mu\underbrace{\left(g\frac{g'}{\sqrt{g^2+2g'^2}}-g'\frac{g}{\sqrt{g^2+2g'^2}}\right)}_{=0}\nu_\ell = 0.
\end{aligned}$$

This proves that the neutrino has zero electric charge, such as we showed in the previous section when we consider the case $v_L=0$ .

**For the case of charged leptons:**

$$\begin{aligned}
\mathcal{L}^{lep}_{\ell_\alpha} &= i\,\overline{\ell}_L\left(\partial\!\!\!/-i\frac{g}{2}\gamma^\mu W^L_{3\mu}-i\frac{g'}{2}\gamma^\mu B_\mu\right)\ell_L+i\,\overline{\ell}_R\left(\partial\!\!\!/-i\frac{g}{2}\gamma^\mu W^R_{3\mu}-i\frac{g'}{2}\gamma^\mu B_\mu\right)\ell_R\\
&= i\,\overline{\ell}_L\;\partial\!\!\!/\ell_L+i\;\underbrace{(-1)\overline{\ell}_L\left(i\frac{g}{2}\gamma^\mu W^L_{3\mu}+i\frac{g'}{2}\gamma^\mu B_\mu\right)\ell_L}_{(I')}+i\,\overline{\ell}_R\;\partial\!\!\!/\ell_R+\\
&+\;i\;\underbrace{(-1)\overline{\ell}_R\left(i\frac{g}{2}\gamma^\mu W^R_{3\mu}+i\frac{g'}{2}\gamma^\mu B_\mu\right)\ell_R}_{(II')}
\end{aligned}$$

considering the interaction with the photon:

$$
\begin{aligned}
(I') + (II') &\overset{A_\mu}{=} \frac{-i}{2} \underbrace{\bar{\ell}_L}_{\bar{\ell} P_R} \gamma^\mu A_\mu \left( g \frac{g'}{\sqrt{g^2 + 2g'^2}} + g' \frac{g}{\sqrt{g^2 + 2g'^2}} \right) \underbrace{\ell_L}_{P_L \ell} \\
&+ \frac{-i}{2} \underbrace{\bar{\ell}_R}_{\bar{\ell}\, PL} \gamma^\mu A_\mu \left( g \frac{g'}{\sqrt{g^2 + 2g'^2}} + g' \frac{g}{\sqrt{g^2 + 2g'^2}} \right) \underbrace{\ell_R}_{P_R\, \ell} \\
&= \frac{-i}{2} \bar{\ell} P_R \gamma^\mu A_\mu \left( g \frac{g'}{\sqrt{g^2 + 2g'^2}} + g' \frac{g}{\sqrt{g^2 + 2g'^2}} \right) P_L \ell \\
&+ \frac{-i}{2} \bar{\ell}_\ell P_L \gamma^\mu A_\mu \left( g \frac{g'}{\sqrt{g^2 + 2g'^2}} + g' \frac{g}{\sqrt{g^2 + 2g'^2}} \right) P_R \ell \\
&= \frac{-i}{2} \bar{\ell} \gamma^\mu A_\mu \underbrace{\left( g \frac{g'}{\sqrt{g^2 + 2g'^2}} + g' \frac{g}{\sqrt{g^2 + 2g'^2}} \right)}_{=2 \frac{g' g}{\sqrt{g^2 + 2g'^2}}} \ell.
\end{aligned}
$$

therefore, we have the electric charge for charged leptons:

$$
Q_e = \frac{-g' g}{\sqrt{g^2 + 2g'^2}},
$$

This result have been done regarding $v_L \neq 0$. A similar result is obtained regarding $v_L = 0$, see that in the appendix G. To mention, this choice is importante because we have the freedom of choosing $v_L = 0$. Remmeber that $\chi_L$ doesn't coupling with charged leptons, and it could be regarded dark matter particle candidate.

Hence, the electric charge is given by:

$$
Q^f = q \frac{g\, g'}{\sqrt{g^2 + 2\, g'^2}}, \tag{3.74}
$$

with $q = -1, 2/3, -1/3$ for charged leptons, u-like and d-like quarks, respectively. Moreover, $Q^\nu = 0$.

From (3.74) is obtained:

$$
\frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2}, \qquad \frac{1}{g_Y^2} = \frac{1}{g^2} + \frac{1}{g'^2}, \tag{3.75}
$$

as was mentioned in the chatper two, see the equation (2.8).

### 3.3.2 Coupling with the Charged Weak Bosons:

**In the lepton sector:**

The lagrangian interaction between charged bosons and leptons can be write in the form:

$$
\mathcal{L}_W = -\frac{g}{2} \left[ \bar{\nu}_L \gamma^\mu V_{PMNS} \ell_L\, W^+_{L\mu} + \bar{\nu}_R \gamma^\mu V_{PMNS} \ell_R\, W^+_{R\mu} \right] + H.C., \tag{3.76}
$$

the mixing matrix is the same in both, left- and right - sectors because the charged lepton and neutrino mass matrices are diagonalized by the same unitary matrices.

In addition, we can write the charged current interactions, in the mass eigenstates basis, introducing a phase factor like a general case, such that the Lagrangian is given by:

$$\begin{aligned} \mathcal{L}_W^l &= -\frac{g}{2}\left[e^{i\phi_l}\overline{\nu_L}\gamma^\mu V_l l_L W^+_{L\mu} + \overline{\nu_R}\gamma^\mu V_l l_R W^+_{R\mu}\right] + H.c. \\ &= -\frac{g}{2}\left[(e^{i\phi_l}c_\xi J_L^{l\mu} + s_\xi J_R^{l\mu})W^+_{1\mu} + (-e^{i\phi_l}s_\xi J_L^{l\mu} + c_\xi J_R^{l\mu})W^+_{2\mu}\right] + H.c., \end{aligned} \tag{3.77}$$

and we have used Eq. (3.19); here $J_L^{l\mu} = \overline{\nu_L}\gamma^\mu V_l l_L$ and $J_R^{l\mu} = \overline{\nu_R}\gamma^\mu V_l l_R$.

**In the quark sector:**

$$\begin{aligned} \mathcal{L}_W^q &= -\frac{g}{2}\left[e^{i\phi_q}\bar{u}_L\gamma^\mu V_{CKM} l_L W^+_{L\mu} + \bar{u}_R\gamma^\mu V_{CKM} d_R W^+_{R\mu}\right] + H.c. \\ &= -\frac{g}{2}\left[(e^{i\phi_q}c_\xi J_L^{q\mu} + s_\xi J_R^{q\mu})W^+_{1\mu} + (-e^{i\phi}s_\xi J_L^{q\mu} + c_\xi J_R^{q\mu})W^+_{2\mu}\right] + H.c., \end{aligned} \tag{3.78}$$

with $J_L^{q\mu} = \bar{u}_L\gamma^\mu V_{CKM} d_L$ and $J_R^{q\mu} = \bar{u}_R\gamma^\mu V_{CKM} d_R$ with $V_{CKM}$ being the same as in the left-handed sector with three angles and one physical phase.

In the general case where the Yukawa couplings in Eq. (2.59) aren't hemitics, the right-handed CKM matrix is different from the left-handed one. This case was considered in Ref. [39].

According with the previous paragraph we can say:

- the phase $\phi_l$ and $\phi_q$ introduced in Eqs. (3.77) and (3.78), respectively, needs an explanation.
- The Dirac fields abserve $2n-1$ phases in the mixing matrix for $n$ Dirac fermions, since one is a global phase.
- In the SM, this is enough, because there is only one charged current and the global phase never appears in amplitudes. However, in this sort of model there are also right-handed charged currents and there is a relative global phase between both charged currents. This phase is $\phi_l$ for leptons and $\phi_q$ for quarks.

### 3.3.3 Coupling with Neutral Weak Boson (Neutral Current):

Next, we parametrize the neutral interactions of a fermion $i$ with the $Z_{1\mu}$ and $Z_{2\mu}$ neutral bosons as follow:

$$\mathcal{L}_{NC} = -\frac{g}{\cos\theta}\sum_i \bar{\psi}_i\gamma^\mu\left[(g_V^i - g_A^i\gamma^5)Z_{1\mu} + (f_V^i - f_A^i\gamma^5)Z_{2\mu}\right]\psi_i \tag{3.79}$$

we will define:

$$g_V^f = \frac{1}{2}(a_L^f + a_R^f), \qquad g_A^f = \frac{1}{2}(a_L^f - a_R^f), \tag{3.80}$$

where $a_L^f$ and $a_R^f$ are the couplings of the left- and right- handed components of a fermion $f$.

**Coupling with $Z_1$:**

$$
\begin{aligned}
\mathcal{L}_{\nu_\alpha} &= -\overline{\nu}_L \left( \frac{g}{2}\gamma^\mu \underbrace{W^L_{3\mu}}_{a_2\, Z_{1\mu}} - \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_2\, Z_{1\mu}} \right) \nu_L - \overline{\nu}_R \left( \frac{g}{2}\gamma^\mu \underbrace{W^R_{3\mu}}_{b_2\, Z_{1\mu}} - \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_2\, Z_{1\mu}} \right) \nu_R \\
&= -\frac{1}{2}\overline{\nu}_L\gamma^\mu \left(a_2\, g - c_2\, g'\right)\, Z_{1\mu}\, \nu_L - \frac{1}{2}\overline{\nu}_R\gamma^\mu \left(b_2\, g - c_2\, g'\right)\, Z_{1\mu}\, \nu_R \\
&= -\frac{1}{2}\left[\overline{\nu}\, P_R\gamma^\mu \left(a_2\, g - c_2\, g'\right) Z_{1\mu} P_L\nu + \overline{\nu}\, P_L\gamma^\mu \left(b_2\, g - c_2\, g'\right) Z_{1\mu} P_R\nu\right] \\
&= -\frac{1}{2}\left[\overline{\nu}\, \gamma^\mu\, P_L \left(a_2\, g - c_2\, g'\right) Z_{1\mu} P_L\nu + \overline{\nu}\, \gamma^\mu\, P_R \left(b_2\, g - c_2\, g'\right) Z_{1\mu} P_R\nu\right] \\
&= -\frac{1}{2}\left[\overline{\nu}\, \gamma^\mu\, P_L^2 \left(a_2\, g - c_2\, g'\right) Z_{1\mu}\nu + \overline{\nu}\, \gamma^\mu\, P_R^2 \left(b_2\, g - c_2\, g'\right) Z_{1\mu}\nu\right] \\
&= -\frac{1}{2}\left[\overline{\nu}\, \gamma^\mu\, P_L \underbrace{\left(a_2\, g - c_2\, g'\right)}_{=A_L^{\nu_\ell}} Z_{1\mu}\nu + \overline{\nu}\, \gamma^\mu\, P_R \underbrace{\left(b_2\, g - c_2\, g'\right)}_{=A_R^{\nu_\ell}} Z_{1\mu}\nu\right] \\
&= -\frac{1}{2}\overline{\nu}\, \gamma^\mu \left[\frac{1}{2}(1-\gamma_5)\, A_L^{\nu_\ell} + \frac{1}{2}(1+\gamma_5)\, A_R^{\nu_\ell}\right] Z_{1\mu}\, \nu \\
&= -\frac{1}{2}\overline{\nu}\, \gamma^\mu \left[\frac{A_L^{\nu_\ell} + A_R^{\nu_\ell}}{2} - \frac{A_L^{\nu_\ell} - A_R^{\nu_\ell}}{2}\gamma_5\right] Z_{1\mu}\, \nu = -\frac{g}{2\cos\theta}\overline{\nu}\, \gamma^\mu \left(g_V^{\nu_\ell} - g_A^{\nu_\ell}\, \gamma_5\right) Z_{1\mu}\nu.
\end{aligned}
$$

where:

$$
A_L^{\nu_\ell} \approx g\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}} + \frac{g\, g'^2\sqrt{g^2+g'^2}\,\sqrt{g^2+2g'^2}[K^2g^2 - v_L^2 g'^2]}{v_R^2(g^2+g'^2)^3} \tag{3.81}
$$

$$
A_R^{\nu_\ell} \approx g\,\sqrt{g^2+g'^2}\,\sqrt{g^2+2g'^2}\left[\frac{g^2\, K^2\; - g'^2 v_L^2}{(g^2+g'^2)^2\, v_R^2}\right] \tag{3.82}
$$

$$
\frac{g}{\cos\theta} g_V^\nu = \frac{1}{2}\left(A_L^{\nu_\ell} + A_R^{\nu_\ell}\right) \tag{3.83}
$$

$$
\frac{g}{\cos\theta} g_A^\nu = \frac{1}{2}\left(A_L^{\nu_\ell} - A_R^{\nu_\ell}\right) \tag{3.84}
$$

remember that:

$$
g\, s_\theta \;=\; g'\sqrt{c_{2\theta}} \;\rightarrow\; \sqrt{\frac{g^2+g'^2}{g^2+2\,g'^2}} = c_\theta
$$

Finally:

$$
A_L^{\nu_\ell} \approx \frac{g}{\cos\theta}\left[1 + \frac{g'^2\,(K^2g^2 -\; v_L^2\, g'^2)}{v_R^2(g^2+g'^2)^2}\right] = \frac{g}{\cos\theta} a_L^{\nu_\ell} \tag{3.85}
$$

$$
A_R^{\nu_\ell} \approx \frac{g\,(K^2g^2 -\; v_L^2\, g'^2)}{(g^2+g'^2)\cos\theta\, v_R^2} = \frac{g}{\cos\theta} a_R^{\nu_\ell} \tag{3.86}
$$

$$
g_V^{\nu_\ell} \approx \frac{1}{2} + \frac{g'^2\,(K^2g^2 -\; v_L^2\, g'^2)}{2\, v_R^2(g^2+g'^2)^2} + \frac{K^2g^2 -\; v_L^2\, g'^2}{2(g^2+g'^2)\, v_R^2} \tag{3.87}
$$

$$
g_A^{\nu_\ell} \approx \frac{1}{2} + \frac{g'^2\,(K^2g^2 -\; v_L^2\, g'^2)}{2\, v_R^2(g^2+g'^2)^2} - \frac{K^2g^2 -\; v_L^2\, g'^2}{2(g^2+g'^2)\, v_R^2} \tag{3.88}
$$

1. Summarizing we have for the case of neutrinos:

$$\begin{aligned}
g_V^{\nu_\ell} &\approx \frac{1}{2} + \frac{g'^2\,(K^2 g^2 -\, v_L^2\, g'^2)}{2\, v_R^2 (g^2+g'^2)^2} + \frac{K^2 g^2 -\, v_L^2\, g'^2}{2(g^2+g'^2)\, v_R^2} \\
g_A^{\nu_\ell} &\approx \frac{1}{2} + \frac{g'^2\,(K^2 g^2 -\, v_L^2\, g'^2)}{2\, v_R^2 (g^2+g'^2)^2} - \frac{K^2 g^2 -\, v_L^2\, g'^2}{2(g^2+g'^2)\, v_R^2} \\
a_L^{\nu_\ell} &\approx 1 + \frac{g'^2\,(K^2 g^2 -\, v_L^2\, g'^2)}{v_R^2 (g^2+g'^2)^2}, \\
a_R^{\nu_\ell} &\approx \frac{K^2 g^2 -\, v_L^2\, g'^2}{(g^2+g'^2)\, v_R^2}
\end{aligned} \tag{3.89}$$

we know the following:

$$\frac{g^2}{g^2+g'^2} = \frac{c_{2\theta}}{c_\theta^2}, \qquad\qquad \frac{g'^2}{g^2+g'^2} = t_\theta^2$$

replacing in 3.89, and taking into account the definitions in 3.36, we have:

$$\begin{aligned}
a_L^{\nu_\ell} &\approx 1 + \frac{c_{2\theta}\, t_\theta^2}{c_\theta^2}\left(x - y\,\frac{s_\theta^2}{c_{2\theta}}\right) \\
a_R^{\nu_\ell} &\approx \frac{c_{2\,\theta}}{c_\theta^2}\left(x - y\,\frac{s_\theta^2}{c_{2\theta}}\right) \\
g_V^{\nu_\ell} &\approx \frac{1}{2} + \frac{c_{2\theta}}{2\, c_\theta^4}\left(x - y\,\frac{s_\theta^2}{c_{2\theta}}\right) \\
g_A^{\nu_\ell} &\approx \frac{1}{2} - \frac{c_{2\theta}^2}{2\, c_\theta^4}\left(x - y\,\frac{s_\theta^2}{c_{2\theta}}\right)
\end{aligned} \tag{3.90}$$

Notice that when $v_R \to \infty$ $(x, y \to 0)$, then:

$$\begin{aligned}
a_L^{\nu_\ell} &\to 1, \qquad a_R^{\nu_\ell} \to 0 \\
g_V^{\nu_\ell} &= g_A^{\nu_\ell} \to \frac{1}{2}
\end{aligned} \tag{3.91}$$

These couplings with neutrinos are consistent with those known from the SM.

2. for the case of charged leptons:

$$
\begin{aligned}
\mathcal{L}_{\ell_\alpha} &= \overline{\ell}_L\left(\frac{g}{2}\gamma^\mu \underbrace{W^L_{3\mu}}_{a_2\,Z_{1\mu}} + \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_2\,Z_{1\mu}}\right)\ell_L + \overline{\ell}_R\left(\frac{g}{2}\gamma^\mu \underbrace{W^R_{3\mu}}_{b_2\,Z_{1\mu}} + \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_2\,Z_{1\mu}}\right)\ell_R \\
&= \frac{1}{2}\overline{\ell}_L\gamma^\mu\left(a_2\,g + c_2\,g'\right)\,Z_{1\mu}\,\ell_L + \frac{1}{2}\overline{\ell}_R\gamma^\mu\left(b_2\,g + c_2\,g'\right)\,Z_{1\mu}\,\ell_R \\
&= \frac{1}{2}\left[\overline{\ell}\,P_R\gamma^\mu\left(a_2\,g + c_2\,g'\right)Z_{1\mu}P_L\ell + \overline{\ell}\,P_L\gamma^\mu\left(b_2\,g + c_2\,g'\right)Z_{1\mu}P_R\ell\right] \\
&= \frac{1}{2}\left[\overline{\ell}\,\gamma^\mu\,P_L\left(a_2\,g + c_2\,g'\right)Z_{1\mu}P_L\ell + \overline{\ell}\,\gamma^\mu\,P_R\left(b_2\,g + c_2\,g'\right)Z_{1\mu}P_R\ell\right] \\
&= \frac{1}{2}\left[\overline{\ell}\,\gamma^\mu\,P_L^2\left(a_2\,g + c_2\,g'\right)Z_{1\mu}\ell + \overline{\ell}\,\gamma^\mu\,P_R^2\left(b_2\,g + c_2\,g'\right)Z_{1\mu}\ell\right] \\
&= \frac{1}{2}\left[\overline{\ell}\,\gamma^\mu\,P_L\underbrace{\left(a_2\,g + c_2\,g'\right)}_{=A^\ell_L}Z_{1\mu}\ell + \overline{\ell}\,\gamma^\mu\,P_R\underbrace{\left(b_2\,g + c_2\,g'\right)}_{=A^\ell_R}Z_{1\mu}\ell\right] \\
&= \frac{1}{2}\overline{\ell}\,\gamma^\mu\left[\frac{1}{2}(1-\gamma_5)\,A^\ell_L + \frac{1}{2}(1+\gamma_5)\,A^\ell_R\right]Z_{1\mu}\,\ell \\
&= \frac{1}{2}\overline{\ell}\,\gamma^\mu\left[\frac{A^\ell_L + A^\ell_R}{2} - \frac{A^\ell_L - A^\ell_R}{2}\gamma_5\right]Z_{1\mu}\,\ell = \frac{g}{2\cos\theta}\overline{\ell}\,\gamma^\mu\left(g^{\ell_\alpha}_V - g^{\ell_\alpha}_A\,\gamma_5\right)Z_{1\mu}\ell.
\end{aligned}
$$

where:

$$
\begin{aligned}
A^{\ell_\alpha}_L &\approx \frac{g^3}{g^2+2g'^2}\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}} - \frac{g'^2 g\left(K^2g^2 - v_L^2 g'^2\right)}{v_R^2(g^2+g'^2)^2}\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}} = \frac{g}{c_\theta}\,a^{\ell_\alpha}_L \\
A^{\ell_\alpha}_R &\approx \sqrt{\frac{g^2+2g'^2}{g^2+g'^2}}\left[-\frac{2g\,g'^2}{g^2+2g'^2} + \frac{g(v_L^2\,g'^4 + (-K^2 - v_L^2)g'^2g^2 + K^2g^4)}{(g^2+g'^2)^2\,v_R^2}\right] = \frac{g}{c_\theta}\,a^{\ell_\alpha}_R
\end{aligned}
\tag{3.92}
$$

also

$$
\begin{aligned}
\frac{g}{c_\theta}g^{\ell_\alpha}_V &= \frac{1}{2}\left(A^{\ell_\alpha}_L + A^{\ell_\alpha}_R\right) \\
\frac{g}{c_\theta}g^{\ell_\alpha}_A &= \frac{1}{2}\left(A^{\ell_\alpha}_L - A^{\ell_\alpha}_R\right)
\end{aligned}
\tag{3.93}
$$

then, we can find:

$$
\begin{aligned}
g^{\ell_\alpha}_V &\approx \frac{1}{2}\left(\frac{g^2-2g'^2}{g^2+2g'^2}\right) + \frac{(K^2\,g^2 - v_L^2\,g'^2)(g^2-2g'^2)}{2\,v_R^2(g^2+g'^2)^2}, \\
g^{\ell_\alpha}_A &\approx \frac{1}{2} - \frac{g^2(K^2\,g^2 - v_L^2 g'^2)}{2\,v_R^2(g^2+g'^2)^2},
\end{aligned}
\tag{3.94}
$$

taking into account the following:

$$
\begin{aligned}
\frac{g^2-2g'^2}{g^2+2g'^2} &= 2c_{2\theta} - 1, \\
\frac{g^2-g'^2}{g^2+g'^2} &= \frac{1-4\,s^2_\theta}{c^2_\theta},
\end{aligned}
\tag{3.95}
$$

we finally have:

$$
\begin{aligned}
a_L^{\ell_\alpha} &\approx c_{2\theta} - \frac{t_\theta^2\, c_{2\theta}}{c_\theta^2}\left(x - y\frac{s_\theta^2}{c_{2\theta}}\right) \\
a_R^{\ell_\alpha} &\approx -2\, s_\theta^2 + x\left(t_\theta^2 - \frac{t_\theta^2\, c_{2\theta}}{c_\theta^2} + \frac{c_{2\theta}^2}{c_\theta^4}\right) - y\, t_\theta^2\, \frac{c_{2\theta}}{c_\theta^2} \\
g_V^{\ell_\alpha} &\approx \frac{1}{2}\left(2c_{2\theta} - 1\right)\left\{1 + \frac{c_{2\theta}}{c_\theta^4}\left(x - y\frac{s_\theta^2}{c_{2\theta}}\right)\right\} \\
g_A^{\ell_\alpha} &\approx \frac{1}{2} - \frac{c_{2\theta}^2}{2\, c_\theta^4}\left(x - y\frac{s_\theta^2}{c_{2\theta}}\right).
\end{aligned}
\tag{3.96}
$$

Notice that when $v_R \to \infty$ $(x, y \to 0)$, then:

$$
\begin{aligned}
a_L^{\ell_\alpha} &\to c_{2\theta}, \qquad a_R^{\ell_\alpha} \to -2\, s_\theta^2, \\
g_V^{\ell_\alpha} &\to \frac{1}{2}\left(2c_{2\theta} - 1\right), \qquad g_A^{\ell_\alpha} \to \frac{1}{2}
\end{aligned}
\tag{3.97}
$$

It can be seen that the $g_V^l$ and $g_A^l$ couplings are the same as those of the SM, that is, at low energy it reproduces known results.

In summary, the coupling constants $(v_R \to \infty)$ with the neutron boson $Z_1$ are shown in the following table:

| $f$ | $g'^f_V$ | $g'^f_A$ |
|---|---|---|
| $\ell$ | $c_{2\theta} - \frac{1}{2}$ | $\frac{1}{2}$ |
| $\nu$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

Table 3.1: Coupling constants between leptons and $Z_1$ neutral boson

**Coupling with $Z_2$:**

**a) for the case of neutrinos:**

$$
\begin{aligned}
\mathcal{L}_{\nu_\alpha} &= -\overline{\nu}_L\left(\frac{g}{2}\gamma^\mu \underbrace{W^L_{3\mu}}_{a_3\,Z_{2\mu}} -\frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_3\,Z_{2\mu}}\right)\nu_L - \overline{\nu}_R\left(\frac{g}{2}\gamma^\mu \underbrace{W^R_{3\mu}}_{b_3\,Z_{2\mu}} -\frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_3\,Z_{2\mu}}\right)\nu_R \\
&= -\frac{1}{2}\overline{\nu}_L\gamma^\mu\left(a_3\,g - c_3\,g'\right)\,Z_{2\mu}\,\nu_L - \frac{1}{2}\overline{\nu}_R\gamma^\mu\left(b_3\,g - c_3\,g'\right)\,Z_{2\mu}\,\nu_R \\
&= -\frac{1}{2}\left[\overline{\nu}\,P_R\gamma^\mu\left(a_3\,g - c_3\,g'\right)Z_{2\mu}P_L\nu + \overline{\nu}\,P_L\gamma^\mu\left(b_3\,g - c_3\,g'\right)Z_{2\mu}P_R\nu\right] \\
&= -\frac{1}{2}\left[\overline{\nu}\,\gamma^\mu\,P_L\left(a_3\,g - c_3\,g'\right)Z_{2\mu}P_L\nu + \overline{\nu}\,\gamma^\mu\,P_R\left(b_3\,g - c_3\,g'\right)Z_{2\mu}P_R\nu\right] \\
&= -\frac{1}{2}\left[\overline{\nu}\,\gamma^\mu\,P_L^2\left(a_3\,g - c_3\,g'\right)Z_{2\mu}\nu + \overline{\nu}\,\gamma^\mu\,P_R^2\left(b_3\,g - c_3\,g'\right)Z_{2\mu}\nu\right] \\
&= -\frac{1}{2}\left[\overline{\nu}\,\gamma^\mu\,P_L\underbrace{\left(a_3\,g - c_3\,g'\right)}_{=A_L'^{\nu_\ell}}Z_{2\mu}\nu + \overline{\nu}\,\gamma^\mu\,P_R\underbrace{\left(b_3\,g - c_3\,g'\right)}_{=A_R'^{\nu_\ell}}Z_{2\mu}\nu\right] \\
&= -\frac{1}{2}\overline{\nu}\,\gamma^\mu\left[\frac{1}{2}(1-\gamma_5)\,A_L'^{\nu_\ell} + \frac{1}{2}(1+\gamma_5)\,A_R'^{\nu_\ell}\right]Z_{2\mu}\,\nu \\
&= -\frac{1}{2}\overline{\nu}\,\gamma^\mu\left[\frac{A_L'^{\nu_\ell}+A_R'^{\nu_\ell}}{2} - \frac{A_L'^{\nu_\ell}-A_R'^{\nu_\ell}}{2}\gamma_5\right]Z_{2\mu}\,\nu = -\frac{g}{2\,c_\theta}\overline{\nu}\,\gamma^\mu\left(f_V'^{\nu_\ell} - f_A'^{\nu_\ell}\,\gamma_5\right)Z_{2\mu}\nu.
\end{aligned}
$$

In the same way of the previous section:

$$
\begin{aligned}
A_L'^{\nu_\ell} &\approx \frac{g'^2}{\sqrt{g^2+g'^2}} - \frac{g^2\sqrt{g^2+g'^2}}{(g^2+g'^2)^3 v_R^2}\left[g^4K^2 + g^2g'^2(2K^2 - v_L^2) - 2v_L^2 g'^4\right] = a_L'^{\nu_\ell}\,\frac{g}{c_\theta}, \\
A_R'^{\nu_\ell} &\approx \sqrt{g^2+g'^2} = a_R'^{\nu_\ell}\,\frac{g}{c_\theta},
\end{aligned}
\tag{3.98}
$$

as a function of $\theta$ angle:

$$
\begin{aligned}
a_L'^{\nu_\ell} &\approx \frac{s_\theta^2}{\sqrt{c_{2\theta}}} - \frac{\sqrt{c_{2\theta}}}{c_\theta^4}\left[x\,c_{2\theta}^2 + (2x-y)\,c_{2\theta}\,s_\theta^2 - 2\,y\,s_\theta^4\right], \\
a_R'^{\nu_\ell} &\approx \frac{c_\theta^2}{\sqrt{c_{2\theta}}},
\end{aligned}
\tag{3.99}
$$

then:

$$
\begin{aligned}
f_V'^{\nu_\ell} &\approx \frac{1}{2}\left\{\frac{1}{\sqrt{c_{2\theta}}} - \frac{\sqrt{c_{2\theta}}}{c_\theta^4}\left[x\,c_{2\theta}^2 + (2x-y)\,c_{2\theta}\,s_\theta^2 - 2\,y\,s_\theta^4\right]\right\}, \\
f_A'^{\nu_\ell} &\approx \frac{1}{2}\left\{-\sqrt{c_{2\theta}} - \frac{\sqrt{c_{2\theta}}}{c_\theta^4}\left[x\,c_{2\theta}^2 + (2x-y)\,c_{2\theta}\,s_\theta^2 - 2\,y\,s_\theta^4\right]\right\},
\end{aligned}
\tag{3.100}
$$

Notice that when $v_R \to \infty$ $(x, y \to 0)$, then:

$$\begin{aligned}
a_L^{\prime \nu_{\ell_\alpha}} &\to \frac{s_\theta^2}{\sqrt{c_{2\theta}}}, \\
a_R^{\prime \nu_{\ell_\alpha}} &\to \frac{c_\theta^2}{\sqrt{c_{2\theta}}}, \\
f_V^{\prime \nu_{\ell_\alpha}} &\to \frac{1}{2\sqrt{c_{2\theta}}}, \\
f_A^{\prime \nu_{\ell_\alpha}} &\to -\frac{\sqrt{c_{2\theta}}}{2}.
\end{aligned} \tag{3.101}$$

we can put the neutrinos coupling constants with the neutral boson $Z_2$ in the following table:

| | $a'_L$ | $a'_R$ | $f'_V$ | $f'_A$ |
|---|---|---|---|---|
| $\nu_{\ell_\alpha}$ | $\frac{s_\theta^2}{\sqrt{c_{2\theta}}}$ | $\frac{c_\theta^2}{\sqrt{c_{2\theta}}}$ | $\frac{1}{2\sqrt{c_{2\theta}}}$ | $-\frac{\sqrt{c_{2\theta}}}{2}$ |

Table 3.2: Coupling constants between neutrinos and $Z_2$ neutral boson

**b) For the case of charged leptons:**

$$\begin{aligned}
\mathcal{L}_{\ell_\alpha} &= \bar{\ell}_L \left( \frac{g}{2}\gamma^\mu \underbrace{W_{3\mu}^L}_{a_3\, Z_{2\mu}} + \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_3\, Z_{2\mu}} \right) \ell_L + \bar{\ell}_R \left( \frac{g}{2}\gamma^\mu \underbrace{W_{3\mu}^R}_{b_3\, Z_{2\mu}} + \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_3\, Z_{2\mu}} \right) \ell_R \\
&= \frac{1}{2}\bar{\ell}_L \gamma^\mu \left(a_3\, g + c_3\, g'\right)\, Z_{2\mu}\, \ell_L + \frac{1}{2}\bar{\ell}_R \gamma^\mu \left(b_3\, g + c_3\, g'\right)\, Z_{2\mu}\, \ell_R \\
&= \frac{1}{2}\left[\bar{\ell}\, P_R \gamma^\mu \left(a_3\, g + c_3\, g'\right) Z_{2\mu} P_L \ell + \bar{\ell}\, P_L \gamma^\mu \left(b_3\, g + c_3\, g'\right) Z_{2\mu} P_R \ell\right] \\
&= \frac{1}{2}\left[\bar{\ell}\, \gamma^\mu\, P_L \left(a_3\, g + c_3\, g'\right) Z_{2\mu} P_L \ell + \bar{\ell}\, \gamma^\mu\, P_R \left(b_3\, g + c_3\, g'\right) Z_{2\mu} P_R \ell\right] \\
&= \frac{1}{2}\left[\bar{\ell}\, \gamma^\mu\, P_L^2 \left(a_3\, g + c_3\, g'\right) Z_{2\mu} \ell + \bar{\ell}\, \gamma^\mu\, P_R^2 \left(b_3\, g + c_3\, g'\right) Z_{2\mu} \ell\right] \\
&= \frac{1}{2}\left[\bar{\ell}\, \gamma^\mu\, P_L \underbrace{\left(a_3\, g + c_3\, g'\right)}_{=A_L'^{\ell}} Z_{2\mu} \ell + \bar{\ell}\, \gamma^\mu\, P_R \underbrace{\left(b_3\, g + c_3\, g'\right)}_{=A_R'^{\ell}} Z_{2\mu} \ell\right] \\
&= \frac{1}{2}\bar{\ell}\, \gamma^\mu \left[\frac{1}{2}(1-\gamma_5)\, A_L'^{\ell} + \frac{1}{2}(1+\gamma_5)\, A_R'^{\ell}\right] Z_{2\mu}\, \ell \\
&= \frac{1}{2}\bar{\ell}\, \gamma^\mu \left[\frac{A_L'^{\ell} + A_R'^{\ell}}{2} - \frac{A_L'^{\ell} - A_R'^{\ell}}{2}\gamma_5\right] Z_{2\mu}\, \ell = \frac{g}{2\, c_\theta}\bar{\ell}\, \gamma^\mu \left(f_V'^{\ell_\alpha} - f_A'^{\ell_\alpha}\, \gamma_5\right) Z_{2\mu} \ell.
\end{aligned}$$

where:

$$\begin{aligned}
A_L'^{\ell_\alpha} &\approx -\frac{g'^2}{\sqrt{g^2+g'^2}} - \frac{g^4\sqrt{g^2+g'^2}(K^2 g^2 - v_L^2 g'^2)}{(g^2+g'^2)^3 v_R^2} = \frac{g}{c_\theta} a_L'^{\ell_\alpha} \\
A_R'^{\ell_\alpha} &\approx \frac{(g^2-g'^2)\sqrt{g^2+g'^2}}{g^2+g'^2} + \frac{2g^2 g'^2(K^2 g^2 - v_L^2 g'^2)}{\sqrt{g^2+g'^2}(g^2+g'^2)^2 v_R^2} = \frac{g}{c_\theta} a_R'^{\ell_\alpha}
\end{aligned} \tag{3.102}$$

Knowing that:

$$g\, s_\theta \;=\; g' \sqrt{c_{2\theta}}$$

we will have:

$$\begin{aligned}
a_L^{\prime \ell_\alpha} &\approx \frac{1}{\sqrt{c_{2\theta}}} \left\{ -s_\theta^2 - \frac{c_{2\theta}^3}{c_\theta^4} \left( x - y \frac{s_\theta^2}{c_{2\theta}} \right) \right\} \\
a_R^{\prime \ell_\alpha} &\approx \frac{1}{\sqrt{c_{2\theta}}} \left\{ 1 - 3\, s_\theta^2 - \frac{(1 - c_{2\,\theta})\, c_{2\theta}^2}{c_\theta^4} \left( x - y \frac{s_\theta^2}{c_{2\theta}} \right) \right\} \\
f_V^{\prime \ell_\alpha} &\approx \frac{1}{2\sqrt{c_{2\theta}}} \left\{ 1 - 4\, s_\theta^2 - \frac{c_{2\theta}^2}{c_\theta^4} \left( x - y \frac{s_\theta^2}{c_{2\theta}} \right) \right\} \\
f_A^{\prime \ell_\alpha} &\approx \frac{1}{2\sqrt{c_{2\theta}}} \left\{ -1 + 2\, s_\theta^2 - \frac{(2\, c_{2\,\theta} - 1)\, c_{2\theta}^2}{c_\theta^4} \left( x - y \frac{s_\theta^2}{c_{2\theta}} \right) \right\}.
\end{aligned} \tag{3.103}$$

Notice that when $v_R \;\to\; \infty$ $(x, y \;\to\; 0)$, then:

$$\begin{aligned}
a_L^{\prime \ell_\alpha} &\to \frac{-s_\theta^2}{\sqrt{c_{2\theta}}}, \\
a_R^{\prime \ell_\alpha} &\to \frac{1 - 3\, s_\theta^2}{\sqrt{c_{2\theta}}}, \\
f_V^{\prime \ell_\alpha} &\to \frac{1 - 4\, s_\theta^2}{2\sqrt{c_{2\theta}}}, \\
f_A^{\prime \ell_\alpha} &\to \frac{-1 + 2\, s_\theta^2}{2\sqrt{c_{2\theta}}}.
\end{aligned} \tag{3.104}$$

In summary, the coupling constants ($v_R \;\to\; \infty$) with the neutron boson $Z_2$ are shown in the following table:

| $f$ | $f_V^{\prime f}$ | $f_A^{\prime f}$ |
|---|---|---|
| $\ell$ | $\frac{1-4\, s_\theta^2}{2\sqrt{c_{2\theta}}}$ | $\frac{-1+2\, s_\theta^2}{2\sqrt{c_{2\theta}}}$ |
| $\nu$ | $\frac{1}{2\sqrt{c_{2\theta}}}$ | $-\frac{\sqrt{c_{2\theta}}}{2}$ |

Table 3.3: Coupling constants between leptons with $Z_2$ neutral boson

**In the quark Sector:**

The same happens with the coefficients of the quark sector in this limit, that is, regarding $v_R^2 \gg X^2$, where $X$ represents other VEV.

### a) Coupling constant Up and Down quark with $Z_1$

We start with the lagrangian given by:

$$\mathcal{L}_{u,d} \;=\; -\frac{g}{2\cos\theta} \bar{f} \gamma^\mu (g_V^f - g_A^f\, \gamma_5) Z_{1\mu} f, \tag{3.105}$$

where:

| $f$ | $g_V^f$ | $g_A^f$ |
|---|---|---|
| Up | $\frac{1}{2} - \frac{4}{3}\sin^2\theta$ | $\frac{1}{2}$ |
| Down | $-\frac{1}{2} + \frac{2}{3}\sin^2\theta$ | $-\frac{1}{2}$ |

Table 3.4: Coupling quark *up*, *down* with $Z_1$ neutral boson

**b) Coupling constant Up and Down quark with $Z_2$**

The lagrangian is given by:

$$\mathcal{L}_{u,d} = -\frac{g}{2\cos\theta}\bar{f}\gamma^\mu(g_V'^f - g_A'^f\gamma_5)Z_{2\mu}f, \tag{3.106}$$

where:

| $f$ | $g_V'^f$ | $g_A'^f$ |
|---|---|---|
| Up | $\frac{4\cos 2\theta - 1}{6\sqrt{\cos 2\theta}}$ | $-\frac{1}{2}\sqrt{\cos 2\theta}$ |
| Down | $-\frac{1+2\cos 2\theta}{6\sqrt{\cos 2\theta}}$ | $\frac{1}{2}\sqrt{\cos 2\theta}$ |

Table 3.5: Coupling quark *up*, *down* with $Z_2$ neutral boson

In the way that Lagrangians are defined, the couplings of the quarks with the neutral bosons are different, like the case with the leptons. We can say:

- Only when $v_R$ is strictly infinite can we identify, at tree level, the angle $\theta$ with $\theta_W$ of the SM.
- Assuming the measured values $g_V^l = 0.03783 \pm 0.00041$ does not imply a stronger lower bound on $v_R$ and the $W_2$ and $Z_2$ masses, which was obtained from the $M_W/M_Z$ ratio in Eq. (3.62).
- the CMS Collaboration using $W_2 \to B + t$ or $W_2 \to T + b$ [$T$ and $B$ are vectorlike quarks (VLQs)] excluded a $W_2$ with a mass below 1.6 TeV at 95% C.L. assuming equal branching ratios for the $W'$ boson to $tB$ and $bT$ and 50% for each VLQ to $qH$, where $H$ is a neutral scalar [40]. If $T$ and $B$ are the known $t$ and $b$ quarks and assuming $W_2$ with coupling to the SM particles equal to the SM weak coupling constant, masses below 3.15 TeV are excluded at the 95% confidence level [41]

Furthermore, if right-handed gauge bosons decay into a high-momentum heavy neutrino and a charged lepton, the LHC has excluded values of the $W_2 \sim W_R$ smaller than 3.85 TeV for $N_R$ in the mass range 0.11.8 TeV [42]. Of course, if there are no extra quarks like $T$ and $B$ and neither heavy right-handed neutrinos, these restrictions for the mass of $W_R$ are not valid anymore.

Only for illustration, we give the partial width at tree level, neglecting the fermion masses, and with $M_{W_2} = 7.8$ TeV

$$\Gamma(W_2^+ \to l^+\nu) \approx \frac{G_F M_{W_1}^2 M_{W_2}}{6\pi\sqrt{2}} \sim 31.57 \text{ GeV}. \tag{3.107}$$

Adding over all fermions, it means a full width $\Gamma \sim 94.71$ GeV. Compare this with the case of the $W$ of the SM, $\Gamma_W = 2.085 \pm 0.042$ GeV [3] where $l$ denotes any of the charged leptons i.e.,

$l = e, \mu, \tau$ without a sum over them.

For the $Z_2$ and also neglecting the fermion masses we have

$$\Gamma(Z_2 \to f\bar{f}) \simeq N_c \left[(a_L^f)^2 + (a_R^f)^2\right] \frac{G_F M_W^2 M_{Z_2}}{24\pi}, \tag{3.108}$$

for leptons $N_c = 1$ and for quarks $N_c = 3$. In the case of leptonic decay, using the couplings in Eq. (3.104), we have $\Gamma(Z_2 \to \ l^- l^+) \sim 3.79$ GeV for any of the three charged leptons, if $M_{Z_2} = 9.28$ TeV, with $\Gamma(Z \to l^- l^+) \ = \ 83.984 \pm 0.086$ MeV [3].

Notice that scalar doublets $\chi_{L,R}$ do not couple to fermions and we will assume that vacuum alignment is such that $v_L = 0$; therefore, this scalar field does not contribute to the gauge boson masses, and, hence, $\chi_L$ is an inert doublet [43]. In this case, the inert character is protected by the left-right symmetry.

# Chapter 4

# Mixing and Lepton Masses

Here we will obtain, assuming the measured matrix elements of the PMNS matrix, the Yukawa couplings to generate the correct charged lepton and neutrino masses. First, we consider the present case, i.e., two bidoublets, then we briefly discuss the case with three bidoublets.

## 4.1 The Two-Bidoublet Case

We need to solve the following equations:

$$U^{\ell\dagger} M^{\ell} U^{\ell} = \hat{M}^{\ell} \tag{4.1}$$

$$U^{\nu_\ell\dagger} M^{\nu_\ell} U^{\nu_\ell} = \hat{M}^{\nu_\ell} \tag{4.2}$$

where:

- $M^{\ell}$, $M^{\nu_\ell}$, are given by:

$$M^{\nu_\ell} = G\,\frac{k_1}{\sqrt{2}} = \frac{k_1}{\sqrt{2}} \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}, \tag{4.3}$$

$$M^{\ell} = F\,\frac{k_1}{\sqrt{2}} = \frac{k_1}{\sqrt{2}} \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \tag{4.4}$$

- $\hat{M}^{\ell}$ and $\hat{M}^{\nu_\ell}$ are the diagonal matrix for charged leptons and neutrinos respectively. That is:

$$\hat{M}^{\ell} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad \hat{M}^{\nu_\ell} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{4.5}$$

Remember that the PMNS mixing matrix is given by:

$$U_{PMNS} = U^{\ell\dagger} U^{\nu_\ell}. \tag{4.6}$$

We are going to propose a unitary matrix with only three angles for the neutrinos (mixing matrix), like the PMNS matrix known in the literature of neutrinos oscillation, however we begin showing a parametrized unitary matrix with three angles plus a phase, like this:

$$U_\nu = U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-i\,\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}\,e^{i\,\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}\,e^{i\,\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}\,e^{i\,\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}\,e^{i\,\delta} & c_{13}c_{23} \end{pmatrix} \tag{4.7}$$

Note that, when we regard a unitary matrix for the neutrinos like the matrix given in the expression 4.7, then the unitary matrix for charged leptons, $F$, should be a diagonal matrix.

Depending on the values of the lightest neutrino mass, the neutrino mass [44] spectrum can be: The normal mass hiercharchy, the inverse mass hierarchy and the Cuasi degenerate hierarchy.

In appendix J the calculations of the Yukawa couplings are shown considering nonzero phase factors.

### 4.1.1 The Normal Mass Hierarchy (NH)

The neutrinos matrix, using the normal mass hierarchy[1] ($m_1 \ll m_2 < m_3$),in eV, is given by the following expression:

$$\hat{M}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.0086 & 0 \\ 0 & 0 & 0.0506 \end{pmatrix}, \tag{4.8}$$

the coupling matrix for the neutrinos, $G$, is given by:

$$G = \frac{\sqrt{2}}{k_1} U_{PMNS} \hat{M}_\nu U^\dagger_{PMNS} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}, \tag{4.9}$$

[1]Particle Data Group - 2020

where:

$$
\begin{aligned}
G_{11} &= \frac{\sqrt{2}}{k_1}\left(0.0504\, s_{13}^2 + 0.0086 s_{12}^2\, c_{13}^2\right), \\
G_{12} &= \frac{\sqrt{2}}{k_1}\left(0.0086\, s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta}) + 0.0504\, c_{13}s_{13}s_{23}\, e^{-i\delta}\right), \\
G_{13} &= \frac{\sqrt{2}}{k_1}\left(0.0086\, s_{12}c_{13}(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta}) + 0.0504\, c_{13}s_{13}c_{23}\, e^{-i\delta}\right), \\
G_{21} &= \frac{\sqrt{2}}{k_1}\left(0.0086\, s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + 0.0504\, c_{13}s_{13}s_{23}\, e^{i\delta}\right), \\
G_{22} &= \frac{\sqrt{2}}{k_1}\left(0.0086\, (c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta})(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + 0.0504\, c_{13}^2 s_{23}^2\right), \\
G_{23} &= \frac{\sqrt{2}}{k_1}\left(0.0086\, (-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta})(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + 0.0504\, c_{13}^2 c_{23}s_{23}\right), \\
G_{31} &= \frac{\sqrt{2}}{k_1}\left(0.0086\, s_{12}c_{13}(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + 0.0504\, c_{13}s_{13}c_{23}\, e^{i\delta}\right), \\
G_{32} &= \frac{\sqrt{2}}{k_1}\left(0.0086\, (c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta})(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + 0.0504\, c_{13}^2 c_{23}s_{23}\right), \\
G_{33} &= \frac{\sqrt{2}}{k_1}\left(0.0086\, (-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta})(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + 0.0504\, c_{13}^2 c_{23}^2\right),
\end{aligned}
\tag{4.10}
$$

In the case where:

$$
\begin{aligned}
s_{12}^2 &= 0.307, \\
s_{23}^2 &= 0.512, \\
s_{13}^2 &= 0.0218, \\
\delta &= 0, \\
k_1 &= 2\,\mathrm{GeV},
\end{aligned}
\tag{4.11}
$$

we obtain real values of the matrix elements:

$$
\begin{aligned}
G_{11} &= 0.2606 \times 10^{-11}, \\
G_{12} &= 0.5481 \times 10^{-11}, \\
G_{13} &= 0.1474 \times 10^{-11}, \\
G_{21} &= 0.5481 \times 10^{-11}, \\
G_{22} &= 1.9583 \times 10^{-11}, \\
G_{23} &= 1.5418 \times 10^{-11}, \\
G_{31} &= 0.1474 \times 10^{-11}, \\
G_{32} &= 1.5418 \times 10^{-11}, \\
G_{33} &= 1.9671 \times 10^{-11},
\end{aligned}
\tag{4.12}
$$

The neutrinos matrix could also be written in the following form:

$$
\hat{M}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & (\Delta m_{21}^2)^{1/2} & 0 \\ 0 & 0 & |\Delta m_{31}^2|^{1/2} \end{pmatrix},
\tag{4.13}
$$

In general, the coupling matrix elements for the neutrinos, in the Hierarchy normal, are given by:

$$
\begin{aligned}
G_{11} &= \frac{\sqrt{2}}{k_1}\left(|\Delta m^2_{31}|^{1/2}\, s^2_{13} + (\Delta m^2_{21})^{1/2} s^2_{12}\, c^2_{13}\right),\\
G_{12} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta}) + |\Delta m^2_{31}|^{1/2}\, c_{13}s_{13}s_{23}\, e^{-i\delta}\right),\\
G_{13} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, s_{12}c_{13}(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta}) + |\Delta m^2_{31}|^{1/2}\, c_{13}s_{13}c_{23}\, e^{-i\delta}\right),\\
G_{21} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c_{13}s_{13}s_{23}\, e^{i\delta}\right),\\
G_{22} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, (c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta})(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c^2_{13}s^2_{23}\right), \qquad (4.14)\\
G_{23} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, (-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta})(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c^2_{13}c_{23}s_{23}\right),\\
G_{31} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, s_{12}c_{13}(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c_{13}s_{13}c_{23}\, e^{i\delta}\right),\\
G_{32} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, (c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta})(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c^2_{13}c_{23}s_{23}\right),\\
G_{33} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, (-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta})(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c^2_{13}c^2_{23}\right),
\end{aligned}
$$

### 4.1.2 The charged leptons coupling matrix

As was mentioned in the previous sections, the charged leptons matrix coupling have to be diagonal, then:

$$F = \frac{\sqrt{2}}{k_1}\hat{M}_\ell = \frac{\sqrt{2}}{k_1}\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \qquad (4.15)$$

Regarding the following values:

$m_e = 0.5109\, MeV$, $m_\mu = 105.6584\, MeV$, $m_\tau = 1776.86\, MeV$ and $k_1 = 2\, GeV$, we obtain the matrix F:

$$F = \begin{pmatrix} 3.6126\times 10^{-4} & 0 & 0 \\ 0 & 0.0747 & 0 \\ 0 & 0 & 1.2564 \end{pmatrix} \qquad (4.16)$$

### 4.1.3 The Inverse Mass Hierarchy (IH)

The Inverted mass hierarchy[2] ($m_3 \ll m_1 < m_2$) is given by the following matricial expression:

$$\hat{M}_\nu = \begin{pmatrix} 0.0497 & 0 & 0 \\ 0 & 0.0504 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (4.17)$$

[2] Particle Data Group - 2020

The coupling matrix for the neutrinos, $G$, is given by:

$$G = \frac{\sqrt{2}}{k_1} U_{PMNS} \hat{M}_\nu U^\dagger_{PMNS} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}, \tag{4.18}$$

Taking into account the numerical values of the mixing angles and the phase angle written in the PDG:

$$s^2_{12} = 0.297 \tag{4.19}$$
$$s^2_{23} = 0.425 \tag{4.20}$$
$$s^2_{13} = 0.0215 \tag{4.21}$$
$$\delta = 0 \tag{4.22}$$
$$k_1 = 2\,\text{GeV}, \tag{4.23}$$

we obtain the following values:

$$\begin{aligned} G_{11} &\approx 3.4531 \times 10^{-11} \\ G_{12} &\approx -0.3530 \times 10^{-11} \\ G_{13} &\approx -0.3762 \times 10^{-11} \\ G_{21} &\approx -0.3530 \times 10^{-11} \\ G_{22} &\approx 1.7677 \times 10^{-11} \\ G_{23} &\approx -1.7352 \times 10^{-11} \\ G_{31} &\approx -0.3752 \times 10^{-11} \\ G_{32} &\approx -1.7352 \times 10^{-11} \\ G_{33} &\approx 1.8568 \times 10^{-11} \end{aligned} \tag{4.24}$$

### 4.1.4 The Quasidegenerate Case (QD)

The Inverted mass hierarchy[3] ($m_1 \cong m_2 \cong m_3 \cong m_0$, $m_0 \gtrsim 0.10\,eV$) is given by the following matricial expression:

$$\hat{M}_\nu = \begin{pmatrix} 0.10 & 0 & 0 \\ 0 & 0.10 & 0 \\ 0 & 0 & 0.10 \end{pmatrix} \tag{4.25}$$

The coupling matrix for the neutrinos, $G$, is given by:

$$G = \frac{\sqrt{2}}{k_1} U_{PMNS} \hat{M}_\nu U^\dagger_{PMNS} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}, \tag{4.26}$$

Taking into account the numerical values of the mixing angles (PDG):

$$\begin{aligned} s^2_{12} &= 0.297 \\ s^2_{23} &= 0.425 \\ s^2_{13} &= 0.0215 \\ \delta &= 0 \\ k_1 &= 2\,\text{GeV}, \end{aligned}$$

[3] Particle Data Group - 2020

we obtain the following values of the Yukawa couplings:

$$\begin{aligned}
G_{11} &= 7.0711\times10^{-11}\\
G_{12} &= -6.1332\times10^{-27}\\
G_{13} &= -1.2266\times10^{-27}\\
G_{21} &= -6.1332\times10^{-27}\\
G_{22} &= 7.0711\times10^{-11}\\
G_{23} &= 4.9065\times10^{-27}\\
G_{31} &= -2.4533\times10^{-27}\\
G_{32} &= -4.9064\times10^{-27}\\
G_{33} &= 7.0711\times10^{-11}
\end{aligned} \tag{4.27}$$

According to these coefficients we can approximate these values as follows (up to a factor $10^{-11}$)

$$G_{11} = G_{22} = G_{33} \approx 7.0711, \tag{4.28}$$

in this case, all the other $G$'s vanish for all practical purposes.

The Yukawa couplings from this model with two bidoublets contain a fine adjustment as in the SM, which would be avoided if we introduce a third bidoublet, being this the price to pay for having Dirac neutrinos and only the known charged leptons plus the right-handed neutrinos. However, we will show that when a third bidoublet is considered, it seems possible to avoid a fine-tuning in the lepton masses.

## 4.2 Describing the case when a third Bi-doublet is introduced

A possible way to avoid fine tuning in our model with two bi-doublets is to introduce a third doublet in order to give mass to the neutrinos with a single vev, although the information that will be given is very superficial since this would correspond to a topic for a future work.

It is interesting that one of the natural hierarchy in field theories are those in which the VEVs are responsible by the spontaneously breaking of symmetries. This is because their values depend on the vacuum alignment and heavy scalars may have small VEVs. Probably this was first noted by Ma [45] and we have seen an example in section 3.1.1, in the case of $k'_1$. Moreover, as we have emphasized before, we already do not know the number and sort of scalars and we can think of an extension of the present model in which three bi-doublets (and the two doublets $\chi_{L,R}$) are introduced.

In this case, the sector of the model which is more affected by the existence of a third bi-doublet is the Yukawa one. Let us denote $\Phi_\nu, \Phi_l$ and $\Phi_q$ the three bi-doublets. We denote the respective VEVs $\sqrt{2}\langle\Phi_\nu\rangle = \mathrm{Diag}(k_\nu\ k'_\nu)$, $\sqrt{2}\langle\Phi_l\rangle = \mathrm{Diag}(k_l\ k'_l)$, and $\sqrt{2}\langle\Phi_q\rangle = \mathrm{Diag}(k_q\ k'_q)$.

We introduce the discrete symmetry $D$ under which [46]

$$D: \quad \Phi_\nu, \Phi_l \to -i\Phi_\nu, -i\Phi_l, \quad R'_l \to iR'_l, \tag{4.29}$$

and all the other fields stay invariant under $D$. In this case, the Yukawa interactions are written as

$$\mathcal{L}_Y = \bar{L}'_l(G^\nu\Phi_\nu + G^l\Phi_l)R'_l + \bar{Q}'_L(G^q\Phi_q + F^q\tilde{\Phi}_q)Q'_R + H.c. \tag{4.30}$$

Notice that the $D$ symmetry forbids the interactions like $\bar{L}'_l\tilde{\Phi}_\nu R'_l$ and $\bar{L}'_l\tilde{\Phi}_l R'_l$, where $\tilde{\Phi}=\tau_2\Phi^*\tau_2$. Although it is out of the scope of this work to analyze the scalar potential and its spectra we note that it may be possible to have a vacuum alignment in which VEVs are hierarchically: $k_\nu, k'_\nu, k_l \ll k'_l \ll k_q, k'_q$, then neutrino masses arise from $k_\nu$, and the charged lepton masses from $k'_l$ (these leptons receive a small contributions from $k'_\nu$). In this situation the mass matrices are given by

$$M^\nu \approx G^\nu \frac{k_\nu}{\sqrt{2}}, \quad M^l \approx G^l \frac{k'_l}{\sqrt{2}}, \quad M^u = G^q \frac{k_q}{\sqrt{2}} + F^q \frac{k'^*_q}{\sqrt{2}}, \quad M^d = G^q \frac{k'_q}{\sqrt{2}} + F^q \frac{k^*_q}{\sqrt{2}}, \tag{4.31}$$

where the quark mass matrices are the same as in most LR symmetric models. If $k_\nu \gtrsim \sqrt{\Delta m^2_{31}}$ then all entries of the matrix $G^\nu$ are of order of unity. The same happens in the Yukawa couplings in the charged lepton sector if $k'_l \gtrsim m_\tau$. Recall that hierarchy among VEVs are more easily justified than in the Yukawa couplings. The quark sector follows as usual. An interesting possibility is when an $S_3$ symmetry is introduced. This has been done in the quark sector by Das and Pal [47], and it is possible to do the same in the lepton sector.

We illustrate how hierarchy between the VEVs can arise considering the SM with two scalar doublets with $Y=+1$, $H_i$, $i=1,2$ with $\langle H^0_1\rangle = v$ and $\langle H^0_2\rangle = u$. Introducing the quadratic Hermitian term in the scalar potential $\mu^2_{12}(H^\dagger_1 H_2 + H^\dagger_2 H_1)$ the constraints equations have the form [45]

$$v[\mu^2_1 + \lambda_1 v^2 + (\lambda_3+\lambda_4)u^2] + \mu^2_{12} u = 0, \quad u[\mu^2_2 + \lambda_2 u^2 + (\lambda_3+\lambda_4)v^2] + \mu^2_{12} v = 0, \tag{4.32}$$

where $\lambda_{1,2,3,4}$ are quartic coupling constants. If $\mu^2_1 < 0$, $\mu^2_2 > 0$, and $|\mu^2_{12}| \ll \mu^2_2$, we have

$$v^2 \simeq -\frac{\mu^2_1}{\lambda_1}, \quad u \simeq -\frac{\mu^2_{12} v}{\mu^2_2 + (\lambda_3+\lambda_4)v^2}. \tag{4.33}$$

We see that $u \ll v$ is possible. A similar mechanism may be at work in the present model but the full scalar potential with three bidoublets and two doublets is rather complicated and needs a separately study.

# Chapter 5

# Phenomenological Consequences

Many of the features of the present model[1] are as those in multi-Higgs models. For instance, FCNC mediated by scalars, several $CP$-violating phases, etc. The existence of FCNCs in the scalar sector has several phenomenological consequences, among others, it means that there are contributions to the muon anomaly $\Delta a_\mu$. For instance, taking the present data for the case of the muon $a_\mu = (g_\mu - 2)/2$ anomaly: $\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 288(63)(49) \times 10^{-11}$ which is about $3.7\sigma$ below the experimental value [3]. In the present model there are several possible contributions to $a_\mu$. Just for illustration consider the contribution of a scalar or a pseudoscalar [48]

$$\Delta a_X^\mu(f) = \frac{m_\mu^2}{8\pi^2 m_X^2} |\mathcal{O}|^2 \int_0^1 dx \, \frac{Q_X(x)}{(1-x)(1-\lambda_X^2) + (\epsilon_f \lambda_X)^2 x}, \tag{5.1}$$

with $X = S, A$, $\epsilon_f = m_f/m_\mu$, where $f$ is the fermion in the internal line, $\lambda_X = m_\mu/M_X$ (if $f$ is lepton tau) and $Q_S(x) = x^2(1+\epsilon - x)$ for a scalar $S$, and $Q_A = x^2(1-\epsilon-x)$ for a pseudoscalar $A$; $\mathcal{O}$ denotes a matrix element in the scalar or pseudoscalar sector and we use, for simplicity, $|\mathcal{O}| = 1$. In order to fit the muon and electron $g-2$ anomalies [49] we need $m_S \gtrsim 4.318$ TeV and $m_A \gtrsim 4.321$ TeV [50] (this reference deals with a 331 model where it is not possible simultaneously resolve such anomalies except for certain conditions that must satisfy the heavy charged leptons, which as is known, our model does not contain such exotic leptons). However, lower masses are allowed if we consider the contributions of all scalar and pseudoscalars in the model. We recall that vector boson contributions $W_1$ and $W_2$ are suppressed by neutrino masses and by the unitarity of the PMNS mixing matrices, where neutral vector bosons have diagonal interactions with leptons.

Below, we will consider mainly the difference with the case of the model with Majorana neutrinos (with triplets in the scalar sector), in particular, when heavy neutrinos do exist, with the present model with Dirac neutrinos.

(i) In the present case, there are no heavy neutrinos that can decay at tree level into a Higgs boson plus an active neutrino, $\nu_R \to H + \nu_L'$. These processes are kinematically forbidden since neutrinos are the lightest particles in the model.

(ii) Flavour-changing lepton number processes as $\mu \to e + \gamma$ and $\mu - e$ are suppressed because of the small neutrino masses. The case $\mu \to ee\bar{e}$ is discussed below. For instance, $\mu \to e + \gamma$ may occur through charged scalar or a $W^+$ where the branching ratio would (up to

[1] Dirac neutrinos in a SU(2) left right symmetric model. Phys. Rev. D102, 075006 (2020)

numerical factors $\sim \alpha$) be proportional to

$$\left|\sum_{k=1}^{3}(V_l^*)_{ik}(V_l)_{jk}\frac{m^2_{\nu_k}}{M^2_W}\right|^2, \tag{5.2}$$

where $V_l$ is the PMNS matrix in Eq. (2.64) and we have used the values given in Eq. (4.11); $M_W$ denotes the mass of $W_L$ or $W_R$. Plugging in numbers, we obtain branching ratios smaller than $10^{-48}$. The suppression is due the small neutrino masses and the GIM suppression factor (unitarity of the matrix $V$). Also, in the present model there is not the (logarithmic) enhancement produced by the doubly charge scalar bosons [51, 52].

(iii) In the present model with Dirac neutrinos, there is no neutrinoless double beta decay $(\beta\beta)_{0\nu}$ and other $|\Delta L| = 2$ processes; the muon decay proccess $\mu \to ee\bar{e}$ are produced at tree level only by neutral scalars, as can be seen from the Yukawa interactions in Eq. (2.65). Among the flavor violation charged lepton decays, this is the one which imposes the strongest restriction on the new physics scenarios. Recall that the present and future experiments sensitivities in this decay are $10^{-12}$ and $10^{-16}$, respectively [53]. However, this process is proportional to (up to kinematic factors) $|V_L^\dagger G V_R|^4/m_X^4$, where $V_L$ and $V_R$ are unitary matrices and $G$ are the Yukawa couplings in Eq. (2.65). In the case of two bidoublets, the $G$ entries are as those in Eqs. (4.11), (4.24) and (4.27), and all of them are rather small $\sim 10^{-11}$; hence their contributions are negligible. This is not the case if we consider three bidoublets where there is no fine-tuning in the Yukawa couplings. In this case, the $\mu \to ee\bar{e}$ decay will impose direct constraints on these couplings. However, in this case Yukawa couplings of the order of $10^{-3}$ will suppress the factor $|V_L^\dagger G V_R|^4 < 10^{-12}$ and the decay $\mu \to ee\bar{e}$ can have a rate near to the experimental limit. The latter case deserves a more detailed study.

(iv) Keung-Senjanovic (KS) noted that the LHC offers an exciting possibility of seeing directly both LR symmetry restoration and lepton number violation production of same sign in charged lepton pairs plus jets: $pp \to W_R^+ \to l_R^+ N_L^c \to l_R^+ W_R^- l_R^+ \to l_R^+ l_R^+ jj$ where $(j)$ denotes jets [54].

(v) Instead of jets, we have another charged leptons and one neutrino; the respective trileptons final state has been considered in Ref. [55]. However, if neutrinos are pure Dirac particles, as in the present model, there are no heavy right-handed neutrinos, and, hence, these sort of processes, since one of the vector bosons is $W_R$ and the other $W_L$; it needs a mass insertion in the neutrino internal line, and, hence the amplitude is proportional to the (active) neutrino mass and, for this reason, negligible. Of course, we can introduce triplets in order to have a seesaw type I or II mechanism, but we think that it is still interesting to study pure Dirac neutrinos in LR symmetric models.

Trilepton processes occur in both Majorana and Dirac neutrino cases, and there are subprocesses like the following

$$\begin{array}{lll}
qq' \to W_R^+ \to \nu_R l_{1L}^+ \to W_R^+ l_{1L}^+ l_{2L}^- \to l_{1L}^+ l_{2L}^- l_{3L}^+ N_R, & (a) & \text{M} \\
qq' \to W_L^+ \to \nu_L l_{1R}^+ \to W_L^+ l_{1R}^+ l_{2L}^- \to l_{1R}^+ l_{2L}^- l_{3R}^+ (N_R)^c, & (b) & \text{M} \\
qq' \to W_R^+ \to \nu_R l_{1L}^+ \to W_L^+ l_{1L}^+ l_{2L}^- \to l_{1L}^+ l_{2L}^- l_{3R}^+ \nu_L, & (c) & \text{D} \\
qq' \to W_L^+ \to \nu_L l_{1R}^+ \to W_R^+ l_{1R}^+ l_{2R}^- \to l_{1R}^+ l_{2R}^- l_{3L}^+ \nu_R & (d) & \text{D} \\
qq' \to W_R^+ \to \nu_R l_{1L}^+ \to W_R^+ l_{1L}^+ l_{2R}^- \to l_{1L}^+ l_{2R}^- l_{3L}^+ \nu_R (N_R), & (e) & \text{D,M} \\
qq' \to W_L^+ \to \nu_L l_{1R}^+ \to W_L^+ l_{1R}^+ l_{2L}^- \to l_{1R}^+ l_{2L}^- l_{3R}^+ \nu_L, & (f) & \text{D,M.}
\end{array} \tag{5.3}$$

Above we denote $N_R$ a heavy right-handed neutrino and $\nu_R$ the right-handed component of a Dirac neutrino. Notice that the processes (c) and (d) need a mass insertion or a Yukawa coupling and they are suppressed if neutrinos are pure Dirac. Hence, at least in principle, it is possible to use these processes to distinguish the Dirac from the Majorana case. When processes can occur in both Majorana and Dirac cases, the Dirac case is suppressed by the small neutrino masses. Of course, the Majorana case allows $\Delta L = 2$, and the Dirac case does not.

## 5.1 Breaking parity first

Any model beyond the SM must match with that model at a given energy, say, the $Z$ pole. In the SM coupling constants $g$ and $g_Y$ have different running with the energy. In the case of LR symmetric models, the same happens with $g$ and $g' \equiv g_{B-L}$ as was noted in Ref. [57]. It means that we cannot keep $g_L = g_R$ for all energies, since quantum corrections imply a finite $\Delta g = g_L - g_R \neq 0$. This is due to the fact that both constants feel different degrees of freedom. Hence, it is interesting to search for models with gauge symmetries as in Eq. (2.1) but in which parity is broken spontaneously by nonzero VEVs [57] or softly if quadratic terms in the scalar potential are different $\mu_L^2 \neq \mu_R^2$ as in Ref [5].

Let us consider as in Ref. [57] the possibility that the symmetries in Eq. (2.1) are broken spontaneously but in the following way: First, the parity $\mathcal{P}$ is broken at an energy scale $\mu_P$ by introducing a neutral pseudoscalar singlet, $\eta \sim (\mathbf{1}, \mathbf{1}, 0)$ with $\eta \to -\eta$ under parity and $\langle \eta \rangle = v_\eta$. Then, the doublet $\chi_R$ breaks the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry to $SU(2)_L \otimes U(1)_Y$. The relevant terms in the scalar potential involving the doublets $\chi_L, \chi_R$ and the isosinglet $\eta$ are the following:

$$\mu_\eta^2 \eta^2 + \lambda_\eta \eta^4 + \mu_{LR}^2(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) + f\eta(\chi_L^\dagger \chi_L - \chi_R^\dagger \chi_R) + \lambda'_\eta \eta^2(\chi_L^\dagger \chi_L + \chi^\dagger \chi_R) \subset V. \tag{5.4}$$

At the energy $\mu_{\mathcal{P}}$, $\mu_\eta^2 < 0$ with $\langle \eta \rangle = v_\eta \simeq \mu_{\mathcal{P}}$, and all the other VEVs are still zero. We obtain

$$\mu_L^2 = \mu_{LR}^2 + f v_\eta + \lambda'_\eta v_\eta^2, \quad \mu_R^2 = \mu_{LR}^2 - f v_\eta + \lambda'_\eta v_\eta^2, \tag{5.5}$$

with the singlet VEV $v_\eta = \sqrt{-\mu_\eta^2/2\lambda_\eta}$. Next, if $\mu_R^2 < 0$ and $|\mu_R^2| \ll v_\eta$, we have that $\langle \chi_R \rangle = v_R \neq 0$. This leads to the interesting case in which the $SU(2)_R$ symmetry-breaking scale is induced by the parity-breaking scale as noted in Refs. [57]. It happens also that $g_L \neq g_R$, for energies in the range $v_R < \mu < v_\eta$, and also $V^L_{PMNS} \neq V^R_{PMNS}$, with $V^L_{PMNS} = V_L^{l\dagger} U_L^\nu$, $V^R_{PMNS} = V_R^{l\dagger} U_R^\nu$. In this case we have to consider the most scalar potential involving two or three bidoublets, $\Phi_i$, two doublets $\chi_{L,R}$, and the singlet $\eta$.

# Chapter 6

# Conclusions

1. Since the early 1980s, most phenomenology of the left-right symmetric models includes triplets and Majorana neutrinos [34]. Since then, the model with the following scalar multiplets: one bi-doublet and two triplets was considered the minimal left-right symmetric model. There is no doubt that this proposal was, and still is, well motivated [60]. However, if the neutrinos ultimately turn out to be Dirac particles, all that effort will have been in vain. For this reason we have to pay attention to Dirac neutrinos, even in the context of the left-right symmetric models.

2. As was obtained in the section 2.2.1, our model must match with the SM model at a given energy, that is, according to the mathing condition: $s_\theta^2 < 1/2$, where the weak mixing angle $\theta$ is, in general, different from the $\theta_W$ (Weinberg angle). This implies that the energy scale at which $g_L(\mu) = g_R(\mu)$ must be below the scale at which: $s_\theta^2(\Lambda) = 1/2$, $\mu < \Lambda$. However, it is important to know that in other energy scale we cannot keep $g_L = g_R$ since quantum corrections imply a finite $\Delta g = g_L - g_R \neq 0$, Ref. [57]. This is due to the fact that both constants feel different degrees of freedom.

3. An important result within this work is the hierarchical relationship of the *vevs*. This was developed in section 3.1.1, working firstly with the more general potential and its constrain equations such that by applying certain symmetries such as parity invariance and the symmetry $\mathcal{Z}_2 \otimes \mathcal{Z}_2'$ we obtained simpler constrain equations and, the most important, the hierarchical relationship:
$$v_R \gg k_2 > k_2' \gg k_1 \gg k_1' \gg v_L$$
   An alternative way of obtaining the same simpler constrain equations is to apply the symmetry $Z_5$ to this general potential but it does not give me information about the hierarchy relationship of the *vevs*. This $Z_5$ symmetry is not applied to the whole Lagrangian, only to the potential, therefore our model can leave out with this symmetry.

4. For the case of the charged gauge boson masses, and using the fact that $v_R \gg X$, where $X$ represents the others *vevs*, we obtained:
$$\begin{aligned} \mathcal{M}_{W_1}^2 &= \frac{g^2}{4}\left(K^2 + \frac{v_L^2 + v_R^2}{2} - \sqrt{\Delta}\right) \approx \frac{g^2}{4}\left(K^2 + \frac{v_L^2}{2}\right) \\ \mathcal{M}_{W_2}^2 &= \frac{g^2}{4}\left(K^2 + \frac{v_L^2 + v_R^2}{2} + \sqrt{\Delta}\right) \approx \frac{g^2}{4} v_R^2, \end{aligned} \tag{6.1}$$

where conclude that $M_{W_2} \gg M_{W_1}$, being $M_{W_1} = W^{\pm}$ the charged gauge boson from the SM. The same happens in the case of $M_{Z_2}$.

$$
\begin{aligned}
\mathcal{M}^2_{Z_1} &\approx \frac{g^2}{4\cos^2\theta}\left(K^2 + \frac{v_L^2}{2}\right) \\
\mathcal{M}^2_{Z_2} &\approx \frac{g^2 + g'^2}{4} v_R^2,
\end{aligned}
\tag{6.2}
$$

Using the minimum value of $v_R = 24$TeV, we obtain: $M_{W_2} = 7.835$ TeV, $M_{Z_2} = 9.284$ TeV. Remmember that for the parity invariance in our model we obtain several conditions, among them $g_L = g_R$, at a given energy scale value.

5. Another important result is the value of $\xi < 1.015999 \times 10^{-4}$ rad, that represents the upper limit of the mixing angle between $W_R - W_L$, or in others words $W_1 - W_2$, where according to recent analysis, the experimental limits to the theoretical calculations for the $W_2$ production, the mixing angle $\xi$ is excluded in the range of $10^{-4}$ and $10^{-3}$. See the section 3.2.5.

6. In our model, when we analysis the coupling between the fermions with the electromagnetic field, we obtained the general form of the the electric charge, that is:

$$
Q^f = q\,\frac{g\,g'}{\sqrt{g^2 + 2g'^2}},
$$

where: $q = 0, -1, 23, -1/3$ for the neutrinos, charged leptons, u-like and d-like quarks, respectively. In addition, from the form of the electric charge we obtain:

$$
\frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2},
$$

this result is according to the obtained in others extended models similar to the left-right models[52].

7. In the context of the SM with three right-handed neutrinos the Yukawa couplings have the hierarchy (using the normal hierarchy) $\Delta y_{31} = (\Delta m^2_{31})^{1/2}/v_{SM} \approx 2 \times 10^{-13}$ and $\Delta y_{21} = (\Delta m^2_{21})^{1/2}/v_{SM} \approx 2 \times 10^{-14}$, which we compare with those couplings that in the present model are given in Eq. (4.12). Although the latter values are smaller than the Yukawa sector in the charged lepton sector, see Eq. (4.16), we note that the dispersion in the neutrino Yukawa couplings is in the range $0.25 - 2.2$ (up to a factor of $10^{-11}$). In this model, as in the old left-right symmetric models without scalar triplets and also no extra charged leptons, neutrinos gain arbitrary small masses. Notice, however that the Yukawa couplings are all almost of the same order of magnitude and about 2 order of magnitude larger compared with those in the context of the SM with three right-handed neutrinos. However, we showed how this fine-tuning in the lepton sector can be avoided at the price of introducing a third bidoublet and the discrete $D$ symmetry. In the latter case, all Yukawa couplings in the lepton and quark sector may be of the order of $\mathcal{O}(1)$ if there is a hierarchy in the VEVs of the three bidoublets, however, this will be studied in more detail in a future work.

8. Many of the features of the present model are those in multi-Higgs models, for instance, the existence of FCNCs in the scalar sector, several CP-violating phases, etc. These topics can

be studied in others future works, however, the existence of FCNCs in the scalar sector has several phenomenological consequences, for instance, there are contributions to the muon anomaly (chapter 5) $\Delta a_\mu$, where $a_\mu = \frac{g_\mu - 2}{2}$ is the muon anomalous magnetic moment. Using the equation (5.1) we obtained the condition that the scalar particle or pseudoscalar particle must fulfill in order to fit the muon and the electron $g-2$ anomalies, $m_S \gtrsim 4.318$ TeV and $m_A \gtrsim 4.321$ TeV.

9. In the last appendix, I consider some results obtained when it is proposed a model where the parity is explicitly broken. There is no restoration of parity at high energies. This was also part of a second publication on the Journal of Physics G, whose title is Explicit Parity Violation in $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ models.

# Appendix A

# Symmetries in Particle Physics

In nature there are different types of symmetries, which can be classified into two main groups:

1. Discrete symmetries.
2. Continous symmetries.

Continuous symmetries can be further classified into: space-time symmetries and internal symmetries. We will concentrate here mainly on the transformations of internal symmetry, which in general act simultaneously on the quantum numbers and the space-time coordinates of an initial particle, transforming it into a particle of different quantum numbers and space-time coordinates in the final state, but keeping the same mass.

There are two kinds of internal symmetries:

1. Global symmetry: where the parameters of the transformation do not depend on the space-time coordinates.
2. Local symmetry: where the parameters of the transformation depend explicitly of the space-time coordinates.

Relying on group theory, we know that every symmetry that can be represented by a Lie group is characterized by a number of generators, and the elements of the group can be represented by a unitary transformation, that is:

$$U(\vec{\alpha}) = \exp\left(i\alpha_a T^a\right) \tag{A.1}$$

where $\alpha_a$ are the transformation parameters $U$ and $T^a$, which are the generators of the group in the corresponding representation. These must satisfy the commutation relations:

$$\left[T^a.T^b\right] = if_{abc}T^c \tag{A.2}$$

where $f_{abc}$ are the structure constants of the Lie algebra.

According to this, we can say that global symmetry tranformations must have of (A.1), while the local symmetry transformations have the form of the following expresion:

$$U(\vec{\alpha}) = \exp\left(i\alpha_a(x)\, T^a\right) \tag{A.3}$$

where $\alpha_a$ depends of the space-time coordinates.

During the 20th century, many physicists devoted themselves to the study of the spectrum of particles and the interactions between them, until historically Weinberg, Salam and Glashow postulated that the base internal local symmetry for constructing the electroweak standard model is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

Leptons and quarks in the SM are organized into three families that they have similar characteristics except for their masses. Here, the right fields (R) or the left fields (L) are given in

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, e_R^-, \begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R$$
$$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \mu_R^-, \begin{pmatrix} c \\ s \end{pmatrix}_L, c_R, s_R$$
$$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \tau_R^-, \begin{pmatrix} t \\ b \end{pmatrix}_L, t_R, b_R$$

function of the chirality operator $\gamma_5$:

$$\psi_L = P_L\,\psi, \qquad \psi_R = P_R\,\psi, \tag{A.4}$$

The particles organized in this way are eigenstates of the isospin operator $T_3$ and of the operator $Y$, called hypercharge, which are generators of the groups $SU(2)_L$ and $U(1)_Y$ respectively. These operators determine two quantum numbers $Y$ and $T_3$, related to the electric charge through the Gell Mann-Nishijima relation:

$$Q = T_3 + \frac{Y}{2}, \tag{A.5}$$

## A.1 Gauge Principle

To interpret the gauge principle let us consider a physical system of particles $\psi$ whose dynamics are described by a Lagrangian density $\mathcal{L}$ which is invariant under a global symmetry $U$. If we promote that this symmetry becomes local $U(x)$, we will be transforming the particles and simultaneously generating a theory of interactions.

The procedure to make an invariant theory under local transformations is as follows: by means of a covariant derivative, new boson fields are introduced, called gauge fields, which interact with the $\psi$ field so that the Lagrangian be gauge invariant. The number of gauge fields and the particular characteristics of gauge interactions depend on the symmetry group, being the number of bosons gauge equal to the number of generators of the symmetry group.

## A.2 SSB of the ESM

A simple definition of the phenomenon of SSB is given by:

*A physical system has spontaneously broken symmetry if the interactions that govern the dynamics of the system have such symmetry and the (empty) ground state of the system does not.*

The spontaneous symmetry breaking has repercussions on the dynamics of the system. One of these implications is described by Goldstone's theorem:

*If a field theory has a global symmetry of the Lagrangian which itself is not vacuum symmetry then there must exist a massless scalar or pseudoscalar boson, associated to each generator that does not annihilate the vacuum, and that has the same quantum numbers. These modes are denoted as Nambu-Goldstone bosons or simply Nambu-Goldstone bosons goldstone.*

## A.3 Fundamental Interactions

In quantum mechanics, particles are distinguished into two groups: bosons with integer spin and fermions with half-integer spin. The statistical properties of bosons and fermions are very different. Bosons follow the Bose-Einstein statistic and can be grouped in the same quantum state, while fermions follow the Fermi-Dirac statistic, where two particles with the same quantum numbers cannot be in the same state. As we already mentioned, in the theory field quantum the interactions between particles are described by the exchange of other particles, known as the ¿mediators? of the force. In the ESM all the elementary particles that make up the matter are fermions, while all the elementary particles that mediate or carry the force are bosons.

The fundamental interactions known so far are electromagnetic, weak, strong and gravitational. They are considered fundamental because they cannot be written in terms of other interactions. The electromagnetic force and that of gravity are infinite in scope and have an intensity that decays with the square of the distance. However, there is no quantum theory of gravity, which would imply the existence of a mediator boson, or graviton. Because there is no quantum theory of gravity and the mass of the particles is very small compared to that of macroscopic objects, the SM does not include gravity.

The weak force is chiral. The concept of chirality is related to that of helicity. The helicity of a particle is the projection of the spin in the direction of motion, thus, a particle can be left or right. Although helicity and chirality are only the same in the case of massless particles, the concept of helicity helps to intuitively understand the concept of chirality. In field theory the chirality is an intrinsic property of particles that is related to left and right transformations under the Poincare group. The chirality of the weak interaction is manifested in the fact that only the left particles and the right anti-particles feel it.

The strong force is mediated by gluons, which have a color charge but no electrical charge. As its name implies, it is the strongest of the fundamental forces. This force is very short range.

Atomic nuclei are composed of particles, which until now have not been seen to have a structure and are considered fundamental. These are called quarks and they have peculiar properties. Quarks have a property or quantum number called color. It has nothing to do with the colors we observe with our eyes (or the wavelengths our eyes perceive), it is simply a name for a conserved charge. In experiments to explore the interior of atomic nuclei it became clear that quarks had a quantum number that can take three different states, and that particles composed of quarks, baryons and mesons are always in a neutral combination of this state. That is why they were agreed to give the names of the primary colors.

The gluons also have a color charge and interact with the quarks through the strong force. This is again different than in the case of the electromagnetic force, where the photons have no electrical charge. The mathematical theory that describes the interaction between quarks and gluons is known as quantum chromodynamics or QCD for short. English (quantum chromodynamics). In the processes where quarks intervene, they never appear isolated as we have already said, but when the quarks collide and interact gluons form jets or hadron jets. There are two fundamental types of quarks: up and down. Quarks have an electrical charge, therefore they feel the electromagnetic force. Hadrons, which are classified as baryons (fermions made up of three quarks) and mesons (bosons made up of two quarks), always have an electric charge that is an integer multiple of the charge of the electron. The neutron and the proton are baryons, the constituent quarks of the first are udd and of the second uud. From the fact that the neutron has zero electric charge and the proton +1 we know that individual quarks have fractional electric charge, u-types have 2/3 charge and d-types 1/3. The decay of neutron to proton in nuclear decay tells us that the neutron transforms into a proton and emits an electron and an anti-neutrino from the electron. Since the neutron is made up of quarks this tells us, at a more fundamental level, that a down quark became an up quark, by the exchange of a vector boson $W-$, which then decays into the electron and its anti-neutrino.

Leptons are also fermions and form, together with quarks, all known matter. Leptons also come in six varieties or "flavors"and can be charged, like the electron, or neutral like the neutrino. Charged and neutral leptons form electroweak doublets. As in the case of quarks, the charged and neutral leptons of different doublets are distinguished only by their mass.

However, quarks can decay into other quarks, and neutrinos can change from one type to another. To that part of elementary particle physics that deals with studying the interactions between the different generations is known generically as "flavour physics", and the processes of decay and transformation from one type of fermion to another It is known as flavor changes. The information of the masses and mixtures (processes that change the flavor) of the quarks is contained in the unitary matrix CKM (Cabibbo- Kobayashi-Maskawa). The equivalent information for neutrinos is found in the PMNS (PontecorvoMakiNakagawaSakata) matrix. These matrices parameterize the difference between the quantum state that participates in electroweak interactions and the quantum state that describes the particle propagating freely (mass state), which is a superposition of different flavors.

Electroweak doublets have the same interactions, and in principle, they would be interchangeable if they had the same mass. In this case the symmetry of the flavor would be exact. A small CP symmetry violation is observed in nature. The CP symmetry is the joint action of the sign change in the charge and the sign change in the spatial coordinates (parity). In order to have CP violation in quarks and leptons it is necessary to have three generations of matter. Thus, the CP violation is included in the CKM matrix, although it does not predict its magnitude.

# Appendix B

# Issues that go beyond the SM

**Asymmetry Matter-Antimatter:** To each particle of matter there also corresponds its anti-particle, which has all the opposite quantum numbers, but the same mass. The existence of the anti-particles was predicted by P.A.M. Dirac in developing his relativistic theory of quantum mechanics. In 1932, in a cosmic ray experiment done by C.D. Anderson, anti-electrons were found and called positrons. When a particle and its anti-particle they collide annihilate and what remains are gamma rays (ultra-energetic photons) or other particle and anti-particle pairs. An obligatory question is why is there more matter than anti-matter in our Universe. Particles of anti-matter from the cosmos reach us, but in a much smaller quantity than those of matter. We can also produce in the laboratory, but from astrophysical observations we can infer that the Universe is made mainly of what we call matter and not of anti-matter. This one is known as the problem of baryogenesis, or the creation of baryons (matter) in the Universe. In order to explain how this asymmetry was reached, it is necessary to have a system outside of equilibrium, an initial asymmetry between matter and anti-matter, as well as a charge-parity (CP) symmetry violation. These are known as the Sakharov conditions. to generate baryogenesis. Although the Sakharov model elegantly explains baryogenesis there is not enough baryon asymmetry or CP violation in the SM to be able to explain the dominance of matter over anti-matter in our Universe.

**The Hierarchy Problem:** The mass of fermions varies greatly, the lightest quark, the up, has a mass of about 2,3 MeV, while the top, heaviest and with the same numbers quantum, has a mass of $\sim$ 173 GeV, that is five orders of magnitude difference. The mass of the electron, the lightest charged lepton, is $\sim 0,511$ MeV and that of tau is $\sim 1,78$ GeV, four orders of magnitude larger. This is the hierarchy problem small or mass hierarchy problem. On the other hand, neutrinos, which were thought they had zero mass, actually they have a tiny mass, but different from zero. If of the neutrinos, what has been measured so far in experiments is the difference in the squared masses between pairs of neutrinos. From these bounds it can be deduced that the heavier neutrino cannot have a mass less than 0.04 eV. There is also a bound cosmology that places an upper bound on the sum of the mass of the neutrinos of 1 eV.

This means that there are another five orders of magnitude between the heaviest neutrino and the electron, which makes the mass hierarchy problem even more pronounced.

One explanation for the small mass of neutrinos is that neutrinos acquire their mass by the seesaw mechanism. This implies the existence of very massive particles that have not been observed, right neutrinos. Neutrinos, as their name implies, have zero electrical charge. Being neutral, it is possible that neutrinos are their own anti-particles. If this is the case, it is said to be a Majorana neutrino, if not, it is said to be a Dirac neutrino. If the neutrino is a Majorana particle we can suppose that in addition to the left neutrinos there could be right sterile neutrinos, which do not participate in the weak interaction. The see-saw mechanism assumes the existence of at least two very massive right neutrinos. The diagonalization of the neutrino mass matrix makes the physical states, the eigenvalues of the mass matrix, to be two very massive neutrinos and another very light one, which would be the one observed. The more massive the right ones, the lighter the left ones, hence the name seesaw.

An open problem in particle physics is why hadrons have much more mass than the sum of the masses of their constituent quarks. The sum of the mass of the constituent quarks of a proton or neutron is barely 1% of its total mass. Strong interaction dynamics are assumed to be responsible for the hadron mass, however the exact mechanism is unknown.

**Dark Matter:** The proposal of a class of non-interacting matter was made by Jan Oort and later Fritz Zwicky around 1930, to explain the discrepancy between the mass estimated from the rotation curves of galaxies and that inferred through luminosity. In order for the observed rotation curves to be consistent with the mass of the galaxies, a component of non-visible matter must be added. This type of matter does not emit or absorb light or electromagnetic radiation, or if it does, it is at a minimal level, and its only significant interaction is gravitational, for this reason it was called matter dark Also at larger scales, the need to assume the existence of dark matter is observed to explain the dynamics of large astronomical objects, such as galaxy clusters, where it is inferred that most of the mass comes from dark matter. Over the years the evidence in favor of the matter hypothesis has increased: analysis of the velocities of the members of galaxy clusters, images of gravitational lensing, as well as observations of the Bullet cluster, among others. The Bala cluster is actually two clusters colliding and the observation of this object indicates the existence of two types of matter, ordinary matter, which interacts with each other slowing down the movement in opposite directions and that produces X-ray emission, and another type of matter that does not interact.

The first question that arises from the observation of dark matter is whether it can be one of the already known particles or some particle not yet detected in terrestrial laboratories. To determine what kind of particle it could be, dark matter is divided into three types, depending on its mass and interactions: cold, warm or cold. hot. Hot dark matter is assumed to be ultra-relativistic, with a very small mass, the obvious candidate being the neutrino. However, the structure formation of the Small-scale universe cannot be explained with hot dark matter as the only component. On the other hand cold dark matter is heavy and non-relativistic and the predictions The assumptions made for the structure formation of the Universe are in general agreement with astronomical observations. Warm dark matter has properties that are a mix between those of cold and warm. Mixed hot and cold dark matter models can also be considered. In these cases the amount of hot dark matter it may be only a small percentage of the total. The hypothesis most favored by current observations is that dark matter is cold and consists of weakly interacting massive particles. However, none of the SM particles can be a candidate for cold dark matter, so it would have to be a new or undiscovered new particles until now.

Dark matter makes up 23% of the total mass of the Universe, visible matter (galaxies and intergalactic gas) makes up $4,6\%$, and the rest is energy content. of the Universe is found in the so-called "dark energy". The dark energy hypothesis arose from the observation that our Universe is expanding rapidly. So far it is not known with certainty what is causing this expansion. Among the best accepted proposals are the following: that there is a constant term cosmological , which is a constant energy density intrinsic to space and constant in time, or the quintessence and scalar field models, which can vary of intensity over time. Although the cosmological constant hypothesis is the most favored and forms part of what is known as the cosmological model $\Gamma$, the The difference between the expected value of cosmological measurements and that calculated from models of elementary particles is huge. The expected value from calculations of the vacuum energy in the SM is about 120 orders of magnitude greater than that needed to explain the expansion of the Universe.

# Appendix C

# Irreducible representations of Lie groups $SU(N)$ using Young's diagrams

The use of Young Tableaux becames very useful for dealing with irreducible representations of the discrete symmetric groups and the Lie Groups (continues groups) $SU(N)$.
In our case we will discuss some important points about the Young's Diagrams applie to the Lie groups $SU(N)$ for the cases when $N = 2, 3, 5$.

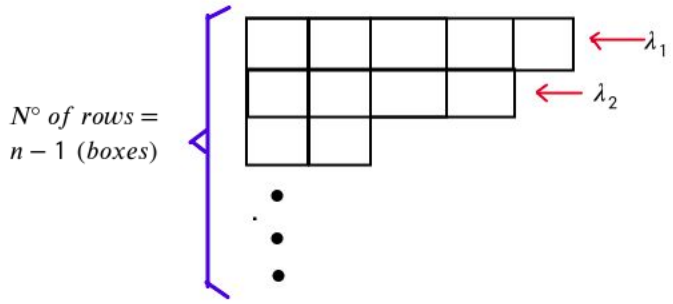


Figure C.1: Every irreps has a particular Young's Diagram

In general:

$$\overline{\lambda} = \left[ \lambda_1, \lambda_2, \cdots, \underbrace{\lambda_n}_{=0} \right], \qquad \lambda_1 \geq \lambda_2 \geq \lambda_3 \cdots \tag{C.1}$$

and the boxes $(SU(n)) = n - 1$. Remember that according to the theory of Young's Tables, it must fulfill that $\lambda_{n-1} \neq 0$, therefore $\lambda_n = 0$, but it is placed for convenience.

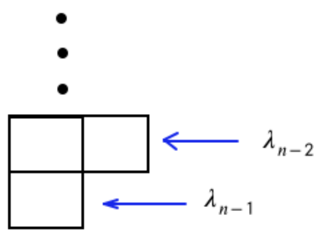


Figure C.2: Following the idea of the above figure

For instance, regarding the case $n = 5$, that is, $SU(5)$, we will have the next diagrams, where the boxes: $n-1 = 5-1 = 4$. where:

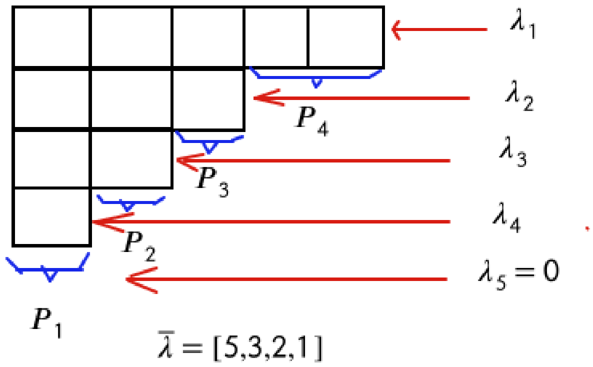


Figure C.3: For the case of $n = 5$, $SU(5)$

$$\overline{\lambda} \;=\; [5, 3, 2, 1]\,, \tag{C.2}$$

or it can also be represented by the number of blocks in the columns ($P_i$). That is:

$$\overline{P} \;=\; (P_1, P_2, P_3 \cdots, P_{n-1})\,, \qquad P_1, P_2, \cdots, P_{n-1} \geq 0, \tag{C.3}$$

Using that in the example, $\overline{P} \;=\; (P_1, P_2, P_3, P_4) = (1, 1, 1, 2)$, where:

$$P_4 = \lambda_1 - \lambda_2; \;\; P_3 = \lambda_2 - \lambda_3; \;\; P_2 = \lambda_3 - \lambda_4; \;\; P_1 = \lambda_4 \tag{C.4}$$

In addition:

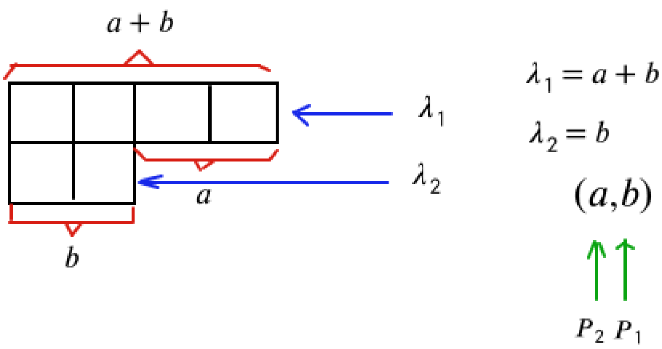


Figure C.4: Notation as a function of $(a, b)$

$$\overline{\lambda} \;=\; [\lambda_1,\, \lambda_2,\, \lambda_3] \equiv \left[\underbrace{\lambda_1 - \lambda_3}_{\lambda'_1}, \underbrace{\lambda_2 - \lambda_3}_{\lambda'_2}, \underbrace{\lambda_3 - \lambda_3}_{\lambda'_3 = 0}\right] \;=\; [\lambda'_1, \lambda'_2, 0] \tag{C.5}$$

they are equivalent when we are talking about the dimension of $SU(N)$, $dim[\overline{\lambda}]$.

$$\dim\left[\overline{\lambda}\right] \;=\; \prod_{i<j}^{n} \frac{(\ell_i - \ell_j)}{(\ell_i^o - \ell_j^o)}, \qquad i, j = 1, 2, \cdots, n \tag{C.6}$$

where:

$$\ell_j^o = n - j; \qquad \ell_j = \lambda_j + n - j$$

1. The case of $SU(2)$: $\bar{\lambda} = [\lambda_1, \lambda_2]$

$$\dim \boxed{\lambda} = \frac{(\ell_1 - \ell_2)}{(\ell_1^o - \ell_2^o)} \tag{C.7}$$

but

$$\begin{aligned} \ell_1^o &= 2 - 1 = 1, \\ \ell_2^o &= 2 - 2 = 0, \\ \ell_1 &= \lambda_1 + 2 - 1 = \lambda_1 + 1, \\ \ell_2 &= \lambda_2 + 2 - 2 = \lambda_2 \end{aligned} \tag{C.8}$$

by replacing in the equation (C.7), we obtain:

$$\dim \boxed{\lambda} = \frac{\lambda_1 + 1 - \lambda_2}{1 - 0} = \underbrace{\lambda_1 - \lambda_2}_{\lambda_1'} + 1 \tag{C.9}$$

hence:

$$\dim \boxed{\lambda} = \frac{\lambda_1 + 1 - \lambda_2}{1 - 0} = \underbrace{\lambda_1 - \lambda_2}_{\lambda_1'} + 1 = \lambda_1' + 1 \tag{C.10}$$

Finally, we obtain:

$$\dim \boxed{\lambda} = \dim(\lambda_1, \lambda_2) = \dim(\lambda_1', 0) = \lambda_1' + 1 \tag{C.11}$$

for example:

$$\begin{aligned} \dim[1, 0] &= 2 = \dim[2, 1], \\ \dim[2, 0] &= 3, \\ \dim[k, 0] &= k, \end{aligned} \tag{C.12}$$

In this Thesis work we have dealt with the gauge group $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$, and we can get the irreps of $SU(2) \otimes SU(2) \equiv \mathbf{2} \otimes \mathbf{2}$, or maybe $SU(2) \otimes SU(2) \otimes SU(2) \equiv \mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2}$

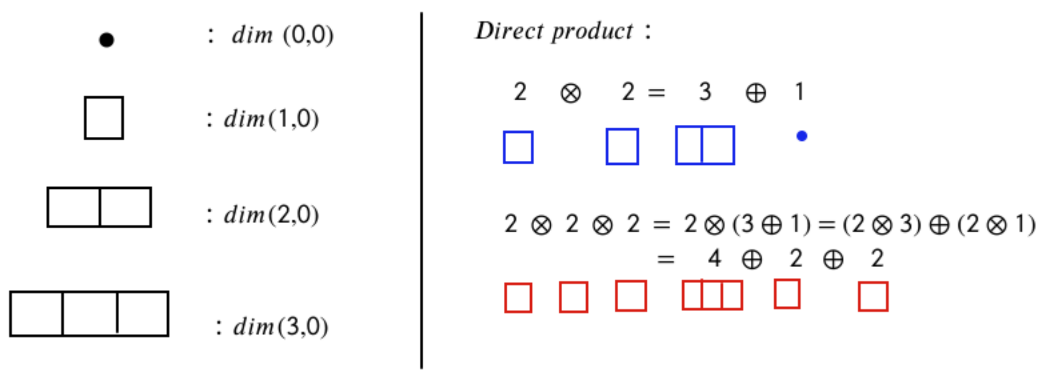


Figure C.5: Young Tableoux's representation $SU(2)$

2. The case of $SU(3)$:
   In the same way that the dimension of $SU(2)$ was obtained.

$$\dim \boxed{\lambda} = \frac{(\ell_1-\ell_2)(\ell_1-\ell_3)(\ell_2-\ell_3)}{(\ell_1^o-\ell_2^o)(\ell_1^o-\ell_3^o)(\ell_2^o-\ell_3^o)} \tag{C.13}$$

   where:

$$\begin{array}{rll} \ell_1 = \lambda_1+3-1 = \lambda_1+2; & \ell_1^o = 3-1 = 2; & \rightarrow \ell_1-\ell_2 = \lambda_1-\lambda_2+1, \\ \ell_2 = \lambda_2+3-2 = \lambda_2+1; & \ell_2^o = 3-2 = 1; & \rightarrow \ell_1-\ell_3 = \lambda_1-\lambda_3+2, \\ \ell_3 = \lambda_3+3-3 = \lambda_3; & \ell_3^o = 3-3 = 0; & \rightarrow \ell_2-\ell_3 = \lambda_2-\lambda_3+1, \end{array} \tag{C.14}$$

   replacing in the equation (C.13), we obtain:

$$\begin{aligned} \dim \boxed{\lambda} &= \frac{(\lambda_1-\lambda_2+1)(\lambda_1-\lambda_3+2)(\lambda_2-\lambda_3+1)}{(1)(2)(1)} \\ &= \frac{(\lambda_1-\lambda_2+1)(\lambda_1-\lambda_3+2)(\lambda_2-\lambda_3+1)}{2} \end{aligned} \tag{C.15}$$

   Finally, we have:

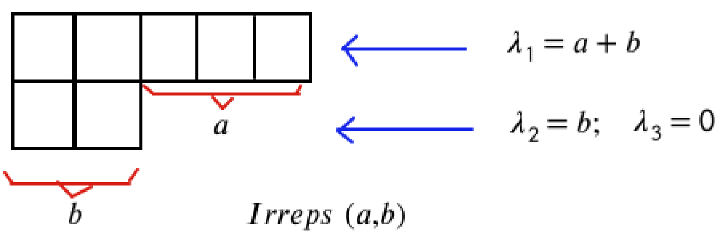


Figure C.6: Blocks in the $SU(3)$

$$\dim \boxed{\lambda} = \dim(\lambda_1,\lambda_2,\lambda_3) = \dim(a,b) = (a+1)(b+1)\left(1+\frac{a+b}{2}\right) \tag{C.16}$$

# Appendix D

# The Minimally extended Standard Model

In the SM, the mass of fermions arises as a result of the Higgs mechanism through the presence of Yukawa couplings of the fermion fields with the Higgs doublet. It is clear that in the SM neutrinos are massless ( their fields do not have a right-handed component) and the Higgs-Lepton Yukawa lagrangian density is given by the following expression:

$$\mathcal{L}_{H,L} \;=\; -\sum_{\alpha,\beta=e,\mu,\tau} Y_{\alpha\beta}\,\overline{L'_{\alpha L}}\Phi\,\ell'_{\beta R} + H.C. \tag{D.1}$$

The quark Yukawa lagrangian density is given by:

$$\mathcal{L}_{H,Q} \;=\; -\sum_{\alpha=1,2,3}\left[\sum_{\beta=d,s,b} Y'^{D}_{\alpha\beta}\,\overline{Q'_{\alpha L}}\Phi\,q'^{D}_{\beta R} + \sum_{\beta=u,c,t} Y'^{U}_{\alpha\beta}\,\overline{Q'_{\alpha L}}\tilde{\Phi}\,q'^{U}_{\beta R}\right] + H.C. \tag{D.2}$$

In the minimally extended SM with three right-handed neutrino fields, The SM Higgs-lepton Yukawa lagrangian in equation(D.1) is extended by adding a lepton term with the same structure as the second term on the right-hand side of equation (D.2), which generates the up-type quarks masses:

$$\mathcal{L}_{H,L} \;=\; -\sum_{\alpha,\beta=e,\mu,\tau} Y'^{\ell}_{\alpha\beta}\,\overline{L_{\alpha L}}\Phi\,\ell'_{\beta R} - \sum_{\alpha,\beta=e,\mu,\tau} Y'^{\nu}_{\alpha\beta}\,\overline{L_{\alpha L}}\Phi\,\nu'_{\beta R} + H.C., \tag{D.3}$$

in the second term, $Y'^{\nu}$, represents a new matriz of Yukawa couplings. Moreover, when the neutral scalar field gains VEV, that is:

$$\langle\Phi\rangle(x) \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{D.4}$$

the lepton Yukawa lagrangian density can be written in matrix form like:

$$\mathcal{L}_{H,L} \;=\; -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\ell'_L}\,Y'^{\ell}\,\ell'_R + \overline{\nu'_L}\,Y'^{\nu}\,\nu'_R\right] + H.C., \tag{D.5}$$

where the arrays of charged lepton fields are given by:

$$\ell'_L \;\equiv\; \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix}, \quad \ell'_R \;\equiv\; \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}, \tag{D.6}$$

and the new right-handed neutrino array:

$$\nu'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}, \tag{D.7}$$

where $Y'^{\ell}$ is the matrix of charged lepton Yukawa couplings that can be diagonalized through the biunitary transformation:

$$V_L^{\ell\dagger} Y'^{\ell} V_R^{\ell} = Y^{\ell}, \qquad \text{with} \qquad Y^{\ell}_{\alpha\beta} = y^{\ell}_{\alpha}\delta_{\alpha\beta} \quad (\alpha, \beta = e, \mu, \tau).$$

The matrix $Y'^{\nu}$ of neutrino Yukawa couplings can be diagonalized in a similar way:

$$V_L^{\nu\dagger} Y'^{\nu} V_R^{\nu} = Y^{\nu}, \qquad \text{with} \qquad Y^{\nu}_{kj} = y^{\nu}_{k}\delta_{kj} \quad (k, j = 1, 2, 3),$$

where $y^{\nu}_k > 0$. $V_L^{\nu}$ and $V_R^{\nu\dagger}$ are two appropiate $3 \times 3$ unitary matrices:

$$V_L^{\nu\dagger} = (V_L^{\nu})^{-1}, \quad \text{and} \quad V_R^{\nu\dagger} = (V_R^{\nu})^{-1}. \tag{D.8}$$

Defining the chiral massive neutrino arrays:

$$n_L = V_L^{\nu\dagger}\,\nu'_L \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}, \qquad n_R = V_R^{\nu\dagger}\,\nu'_R \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}, \tag{D.9}$$

for instance, the term $\overline{\nu'_L}\, Y'^{\nu}\,\nu'_R$ can be written of the following form:

$$\begin{aligned} \overline{\nu'_L}\, Y'^{\nu}\,\nu'_R &= (\nu'_L)^{\dagger}\,\gamma^{o}\,Y'^{\nu}\,\nu'_R \\ &= \overline{n}_L\,\underbrace{(V_L^{\nu})^{-1}}_{V_L^{\nu\dagger}}\,Y'^{\nu}\,\underbrace{\left(V_R^{\nu\dagger}\right)^{-1}}_{V_R^{\nu}}\,n_R \\ &= \overline{n}_L\,\underbrace{\left(V_L^{\nu\dagger}\,Y'^{\nu}\,V_R^{\nu}\right)}_{Y^{\nu}}\,n_R = \overline{n}_L\,Y^{\nu}\,n_R, \end{aligned} \tag{D.10}$$

the diagonalized Higgs-lepton Yukawa lagrangian density reads:

$$\begin{aligned} \mathcal{L}_{H,L} &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\overline{\ell_L}\,Y^{\ell}\,\ell_R + \overline{n_L}\,Y^{\nu}\,n_R\right] + H.C. \\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha=e,\mu,\tau} y^{\ell}_{\alpha}\,\overline{\ell_{\alpha L}}\,\ell_{\alpha R} + \sum_{k=1}^{3} y^{\nu}_{k}\,\overline{\nu_{kL}}\,\nu_{kR}\right] + H.C., \end{aligned} \tag{D.11}$$

the Dirac neutrino fields:

$$\nu_k = \nu_{kL} + \nu_{kR}, \qquad (k = 1, 2, 3), \tag{D.12}$$

finally, we obtain:

$$\mathcal{L}_{H,L} = -\sum_{\alpha=e,\mu,\tau}\frac{y^{\ell}_{\alpha}\,v}{\sqrt{2}}\,\overline{\ell_{\alpha}}\,\ell_{\alpha} - \sum_{k=1}^{3}\frac{y^{\nu}_{\alpha}\,v}{\sqrt{2}}\,\overline{\nu_k}\,\nu_k - \sum_{\alpha=e,\mu,\tau}\frac{y^{\ell}_{\alpha}\,v}{\sqrt{2}}\,\overline{\ell_{\alpha}}\,\ell_{\alpha}\,H - \sum_{k=1}^{3}\frac{y^{\nu}_{\alpha}\,v}{\sqrt{2}}\,\overline{\nu_k}\,\nu_k\,H, \tag{D.13}$$

Furthermore, the neutrino masses are given by:

$$m_k = \frac{y_k^\nu \, v}{\sqrt{2}}, \qquad (k = 1, 2, 3), \tag{D.14}$$

We can observe that the neutrino masses are proportional to the Higgs VEV $v$, like the masses of charged leptons and quarks. However, it is knwon that the masses of neutrinos are much smaller than those of charged leptons and quarks. In this mechanism, there is no explanation of the very small values of the eigenvalues $y_k^\nu$ of the Higgs-neutrino Yukawa coupling matrix which are needed.

# Appendix E

# Majorana Neutrinos

Remember that a chiral fermion fields are building blocks of the SM which is related with the modern gauge theories. It is known that chiral spinors are the smallest irreducible representations of the Lorentz group, from which all representations can be built.
The Dirac equation:

$$(i\,\gamma^{\mu}\partial_{\mu} - m)\;\psi = 0 \tag{E.1}$$

for a fermion field:

$$\psi \;=\; \psi_L + \psi_R, \tag{E.2}$$

where:

$$\begin{aligned}\psi_L = P_L\psi = \frac{1}{2}\,(1+\gamma_5)\;\psi,\\ \psi_R = P_R\psi = \frac{1}{2}\,(1-\gamma_5)\;\psi,\end{aligned}$$

multiplying by $P_R$ to the left of equation (E.3), we obtain:

$$\begin{aligned}P_R \times (i\,\gamma^{\mu}\partial_{\mu} - m)\;\psi &= 0,\\ (i\,P_R\gamma^{\mu}\partial_{\mu} - mP_R)\;\psi &= 0,\\ i\,\gamma^{\mu}P_L\partial_{\mu}\psi - mP_R\,\psi &= 0,\\ i\,\gamma^{\mu}\partial_{\mu}\left(\underbrace{P_L\psi}_{\psi_L}\right) - m\,\underbrace{P_R\,\psi}_{\psi_R} &= 0,\end{aligned}$$

finally we have:

$$i\,\gamma^{\mu}\partial_{\mu}\psi_L = m\psi_R \tag{E.3}$$

In the same way, multiplying by $P_L$ to the left of equation (E.3), we obtain:

$$\begin{aligned}P_L \times (i\,\gamma^{\mu}\partial_{\mu} - m)\;\psi &= 0,\\ (i\,P_L\gamma^{\mu}\partial_{\mu} - mP_L)\;\psi &= 0,\\ i\,\gamma^{\mu}P_R\partial_{\mu}\psi - mP_L\,\psi &= 0,\\ i\,\gamma^{\mu}\partial_{\mu}\left(\underbrace{P_R\psi}_{\psi_R}\right) - m\,\underbrace{P_L\,\psi}_{\psi_L} &= 0,\end{aligned}$$

finally we have:

$$i\,\gamma^\mu \partial_\mu \psi_R = m\psi_L, \tag{E.4}$$

we can see that the chiral fields $\psi_L$ y $\psi_R$, from the equations (E.3) and (E.4), are coupled by the mass $m$.
If we consider that the fermion is massless, the two equations in (E.3) and (E.4) are decoupled:

$$i\,\gamma^\mu \partial_\mu \psi_L \;=\; 0 \tag{E.5}$$
$$i\,\gamma^\mu \partial_\mu \psi_R \;=\; 0 \tag{E.6}$$

Therefore, a massless fermion can be described by a single chiral field (left-handed or right-handed), which has only two independent components. These equation are called the Weyl equations and the spinors $\psi_L$ and $\psi_R$ are called Weyl spinors.
The posibility of describing a physical particle with Weyl spinor was rejected by Pauli in 1933 because it leads to the violation of parity. In addition, space inversion transforms $\psi_L$ into $\psi_R$ and vice versa which implies that parity conservation requires the simultaneous existence of both chiral components, however, the discovery of parity violation in 1956-57 invalidated Pauli's arguments, renewing the posibility to describe massless particles with Weyl spinor fields.
Landau, Lee, Yang and Salam proposed to describe the neutrino with a left-handed Weyl spinor $\nu_L$. This is the so-called two component theory of massless neutrinos, which has been incorporated in the SM, where neutrinos are massless and dscribed by left-handed Weyl spinors.
It is known that a two component spinor is suficient for the description of a massless fermion, however, for the description of a massive particle using a four-component spinor isn't enough. The trick lies in the assumption that $\psi_R$ and $\psi_L$ are not independent. Obviously, the relation connecting $\psi_R$ and $\psi_L$ must be compatible with the equations (E.3) and (E.4), which means that the two equations must be two ways of writing the same equation for one independent field, say $\psi_L$. We can obtain the equation (E.3) from equation (E.4).
Taking the Hermitian conjugate of equation (E.4):

$$\begin{aligned} -i\,\left(\partial_\mu \psi_R\right)^\dagger \gamma^{\mu\dagger} &= m\psi_L^\dagger \\ -i\,\partial_\mu \psi_R^\dagger\, \gamma^{\mu\dagger} &= m\psi_L^\dagger \end{aligned}$$

multiplying it on the right by $\gamma^o$:

$$\left\{ -i\,\partial_\mu \psi_R^\dagger \underbrace{\gamma^{\mu\dagger}}_{\gamma^o\gamma^\mu\gamma^o} = m\psi_L^\dagger \right\} \times \gamma^o$$

reducing this expression:

$$-i\,\partial_\mu \left(\psi_R^\dagger \gamma^o\right) \gamma^\mu \;=\; m\,\left(\psi_L^\dagger\, \gamma^o\right)$$

hence:

$$-i\,\partial_\mu \overline{\psi_R}\, \gamma^\mu \;=\; m\,\overline{\psi_L}, \tag{E.7}$$

in order to obtain the same structure as equation (E.3), we take the transpose of equation (E.7) and multiply on the left with the charge conjugation matrix $\mathcal{C}$, that is:

$$\begin{aligned} -i\,\left(\gamma^\mu\right)^T\,\partial_\mu\,\left(\overline{\psi_R}\right)^T &= m\,\left(\overline{\psi_L}\right)^T, \\ -i\,\mathcal{C}\,\left(\gamma^\mu\right)^T\,\partial_\mu\,\left(\overline{\psi_R}\right)^T &= m\,\mathcal{C}\,\left(\overline{\psi_L}\right)^T \end{aligned} \tag{E.8}$$

using the property of $\mathcal{C}$ into (E.8):

$$\mathcal{C}\,(\gamma_\mu)^T\,\mathcal{C}^{-1} \;=\; -\gamma_\mu, \qquad \rightarrow \qquad \mathcal{C}\,(\gamma_\mu)^T \;=\; -\gamma_\mu\mathcal{C}$$

we have:

$$-i\;(-\gamma^\mu\,\mathcal{C})\;\partial_\mu\overline{\psi_R}^T \;=\; m\,\mathcal{C}\overline{\psi_L}^T \tag{E.9}$$

or equivalently:

$$i\,\gamma^\mu\partial_\mu\,\mathcal{C}\,\overline{\psi_R}^T = m\,\mathcal{C}\,\overline{\psi_L}^T \tag{E.10}$$

This equation has the same structure as equation (E.3), that is, it is identical if we set:

$$\psi_R \;=\; \xi\,\mathcal{C}\,\overline{\psi_L}^T, \tag{E.11}$$

where $\xi$ is an arbitrary phase factor, $|\xi|^2 = 1$. This is the Majorana relation between $\psi_R$ and $\psi_L$, which makes sense because $\mathcal{C}\,\overline{\psi_L}$ is right-handed, that is:

$$P_L\left(\mathcal{C}\,\overline{\psi_L}^T\right) \;=\; P_L\,\mathcal{C}\,\overline{\psi_L}^T, \tag{E.12}$$

but

$$\mathcal{C}\left(\gamma^5\right)^T\mathcal{C}^{-1} \;=\; \gamma^5 \rightarrow \mathcal{C}\gamma^5 = \gamma^5\,\mathcal{C}, \qquad \text{remember that:} \qquad \left(\gamma^5\right)^T = \gamma^5$$

then

$$P_L\mathcal{C} = \frac{1}{2}(1-\gamma^5)\mathcal{C} = \frac{1}{2}\left(\mathcal{C}-\gamma^5\mathcal{C}\right) = \frac{1}{2}\left(\mathcal{C}-\mathcal{C}\gamma^5\right) = \mathcal{C}\frac{1}{2}(1-\gamma^5) = \mathcal{C}\,P_L \tag{E.13}$$

on the equation (E.12):

$$\begin{aligned} P_L\left(\mathcal{C}\,\overline{\psi_L}^T\right) &= \mathcal{C}\,\left(P_L\,\overline{\psi_L}^T\right) = \mathcal{C}\,\left(P_L^T\,\overline{\psi_L}^T\right) = \mathcal{C}\,\left(\overline{\psi_L}\,P_L\right)^T \\ &= \mathcal{C}\,\left(\psi_L^\dagger\gamma^o\,P_L\right)^T = \mathcal{C}\,\left(\psi_L^\dagger\,P_R\,\gamma^o\right)^T = \mathcal{C}\,\left[(P_R\psi_L)^\dagger\;\gamma^o\right]^T \\ &= \mathcal{C}\left[\left(\underbrace{P_R\,P_L}_{=0}\psi\right)^\dagger\gamma^o\right]^T = 0 \end{aligned} \tag{E.14}$$

from equations (E.3) and (E.11) we obtain the Majorana equation for the chiral field:

$$i\,\gamma^\mu\partial_\mu\,\psi_L \;=\; m\,\xi\,\mathcal{C}\,\overline{\psi_L}^T, \tag{E.15}$$

rephasing the field $\psi_L$ in order to eliminate the phase factor $\xi$:

$$\psi_L \;\rightarrow\; \xi\,\psi_L$$

then, the Majorana equation for the chiral field $\psi_L$:

$$i\,\gamma^\mu\partial_\mu\,\psi_L \;=\; m\,\mathcal{C}\,\overline{\psi_L}^T, \tag{E.16}$$

the Majorana condition for the field $\psi$ on the equation (E.2):

$$\psi \;=\; \psi_L + \psi_R \;=\; \psi_L + \mathcal{C}\,\overline{\psi_L}^T \tag{E.17}$$

is given by:

$$\psi = \mathcal{C}\,\overline{\psi}^T \tag{E.18}$$

that is, from equation (E.17):

$$\psi = \psi_L + \psi_R = \psi_L + \mathcal{C}\underbrace{\overline{\psi_L}^T}_{=P_R\,\overline{\psi}^T} = \psi_L + P_R\,\mathcal{C}\,\overline{\psi}^T \tag{E.19}$$

multiplying to the left of (E.20) by $P_R$:

$$P_R\,\psi = \underbrace{P_R\psi_L}_{=0} + \underbrace{P_R^2}_{=P_R}\,\mathcal{C}\,\overline{\psi}^T \tag{E.20}$$

Finally, we obtain that:

$$\psi = \mathcal{C}\,\overline{\psi}^T, \tag{E.21}$$

One can see that $\mathcal{C}\,\overline{\psi}^T$ is identical, apart from a possible phase factor, to the charge conjugated field $\psi_L^C$, that is:

$$\psi_L^C = \mathcal{C}\,\overline{\psi}^T, \tag{E.22}$$

the Majorana field $\psi$ can also be written as:

$$\psi = \psi_L + \psi_L^C, \tag{E.23}$$

and the Majorana condition can also be written like:

$$\psi = \psi^C \tag{E.24}$$

This condition, called Majorana condition, implies the equality of particle and antiparticle. Therefore, only neutral fermions can be described by a Majorana field.

# Appendix F

# The most general Scalar Potential

1. **The complete scalar potential:**

$$V = V^{(2)} + V^{(4a)} + V^{(4b)} + V^{(4c)} + V^{(4d)} + V^{(4e)} \tag{F.1}$$

Where:

$$\begin{aligned}
V^{(2)} &= \frac{1}{2}\sum_{i,j}^{2}\left[\mu_{ij}^2\, Tr(\Phi_i^\dagger\,\Phi_j) + \tilde{\mu}_{ij}^2\, Tr(\tilde{\Phi}_i^\dagger\,\Phi_j) + H.C.\right] + \mu_{LR}^2(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R), && \text{(F.2)}\\
V^{(4a)} &= \frac{1}{2}\sum_{i,j}^{2}\left[\lambda_{ij}\, Tr(\Phi_i^\dagger\,\Phi_j)^2 + \tilde{\lambda}_{ij}\, Tr(\tilde{\Phi}_i^\dagger\,\Phi_j)^2 + H.C.\right], && \text{(F.3)}\\
V^{(4b)} &= \frac{1}{2}\sum_{i,j}^{2}\left[\lambda'_{ij}\,(Tr\Phi_i^\dagger\,\Phi_j)^2 + \tilde{\lambda}'_{ij}\,(Tr\tilde{\Phi}_i^\dagger\,\Phi_j)^2 + H.C.\right], && \text{(F.4)}\\
V^{(4c)} &= \rho_{12}\, Tr(\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2) + \tilde{\rho}_{12}\, Tr(\tilde{\Phi}_1^\dagger\Phi_1\tilde{\Phi}_2^\dagger\Phi_2), && \text{(F.5)}\\
V^{(4d)} &= \frac{1}{2}\left[\sum_{i,j}^{2}(\Lambda_{ij}\, Tr\Phi_i^\dagger\Phi_j + \tilde{\Lambda}_{ij}\, Tr\tilde{\Phi}_i^\dagger\Phi_j)(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)\right.\\
&+ \bar{\Lambda}_{ij}(\chi_L^\dagger\Phi_i\Phi_j^\dagger\chi_L + \chi_R^\dagger\Phi_i^\dagger\Phi_j\chi_R) + \Omega_{ij}(\chi_L^\dagger\tilde{\Phi}_i\Phi_j^\dagger\chi_L + \chi_R^\dagger\tilde{\Phi}_i^\dagger\Phi_j\chi_R)\\
&+ \left.\bar{\Lambda}'_{ij}(\chi_L^\dagger\tilde{\Phi}_i\tilde{\Phi}_j^\dagger\chi_L + \chi_R^\dagger\tilde{\Phi}_i^\dagger\tilde{\Phi}_j\chi_R) + \Omega'_{ij}(\chi_L^\dagger\Phi_i\tilde{\Phi}_j^\dagger\chi_L + \chi_R^\dagger\Phi_i^\dagger\tilde{\Phi}_j\chi_R) + H.C.\right] && \text{(F.6)}\\
V^{(4e)} &= \lambda_{LR}\left[(\chi_L^\dagger\chi_L)^2 + (\chi_R^\dagger\chi_R)^2\right]. && \text{(F.7)}
\end{aligned}$$

2. **The minimum potential:**

(a) The minimum scalar potential $\langle V^{(2)}\rangle$:

$$\begin{aligned}
\langle V^{(2)}\rangle &= \frac{\mu_{11}^2}{2}\left(k_1^2 + k_1'^2\right) + \left(\frac{\mu_{12}^2 + \mu_{21}^2}{2}\right)(k_1k_2 + k_1'k_2') + \frac{\mu_{22}^2}{2}\left(k_2^2 + k_2'^2\right) + \tilde{\mu}_{11}^2\, k_1k_1' +\\
&+ \tilde{\mu}_{22}^2\, k_2k_2' + \left(\frac{\tilde{\mu}_{12}^2 + \tilde{\mu}_{21}^2}{2}\right)(k_1'k_2 + k_1k_2') + \frac{1}{2}\left(v_L^2 + v_R^2\right)\mu_{LR}^2
\end{aligned}$$

(b) The minimum scalar potential $\langle V^{(4a)}\rangle$:

$$\begin{aligned}\langle V^{(4a)}\rangle &= \frac{\lambda_{11}}{4}\left(k_1^4+k_1'^4\right)+\left(\frac{\lambda_{12}+\lambda_{21}}{4}\right)\left(k_1^2k_2^2+k_1'^2k_2'^2\right)+\left(\frac{\tilde{\lambda}_{12}+\tilde{\lambda}_{21}}{4}\right)\left(k_1^2k_2'^2+k_1'^2k_2^2\right)+\\ &+ \frac{\lambda_{22}}{4}\left(k_2^4+k_2'^4\right)+\frac{1}{2}\left(\tilde{\lambda}_{11}\,k_1^2k_1'^2+\tilde{\lambda}_{22}\,k_2^2k_2'^2\right)\end{aligned}$$

(c) The minimum scalar potential $\langle V^{(4b)}\rangle$:

$$\begin{aligned}\langle V^{(4b)}\rangle &= \frac{\lambda_{11}'}{4}\left(k_1^2+k_1'^2\right)^2+\frac{1}{4}\left(\lambda_{12}'+\lambda_{21}'\right)\left(k_1k_2+k_1'k_2'\right)^2+\frac{\lambda_{22}'}{4}\left(k_2^2+k_2'^2\right)^2+\tilde{\lambda}_{11}'k_1^2k_1'^2+\\ &+ \tilde{\lambda}_{22}'k_2^2k_2'^2+\frac{1}{4}\left(\tilde{\lambda}_{12}'+\tilde{\lambda}_{21}'\right)\left(k_1k_2'+k_2k_1'\right)^2\end{aligned}$$

(d) The minimum scalar potential $\langle V^{(4c)}\rangle$:

$$\langle V^{(4c)}\rangle = \frac{\rho_{12}}{4}\left(k_1^2\,k_2^2+k_1'^2\,k_2'^2\right)+\frac{\tilde{\rho}_{12}}{2}\left(k_1k_2k_1'k_2'\right)$$

(e) The minimum scalar potential $\langle V^{(4d)}\rangle$:

$$\begin{aligned}\langle V^{(4d)}\rangle &= \frac{1}{4}\left(v_L^2+v_R^2\right)\left\{\Lambda_{11}\left(k_1^2+k_1'^2\right)+\left(\Lambda_{12}+\Lambda_{21}\right)\left(k_1\,k_2+k_1'\,k_2'\right)+\Lambda_{22}\left(k_2^2+k_2'^2\right)\right\}+\\ &+ \frac{1}{4}\left(v_L^2+v_R^2\right)\left\{\tilde{\Lambda}_{11}k_1k_1'+\left(\tilde{\Lambda}_{12}+\tilde{\Lambda}_{21}\right)\left(k_1'\,k_2+k_1\,k_2'\right)+\tilde{\Lambda}_{22}k_2k_2'\right\}+\\ &+ \frac{(\,v_R^2+v_L^2)}{4}\left\{\overline{\Lambda}_{11}\,k_1'^2+k_1'k_2'\left(\overline{\Lambda}_{12}+\overline{\Lambda}_{21}\right)+\overline{\Lambda}_{22}\,k_2'^2+\overline{\Lambda}_{11}'\,k_1^2+k_1k_2\left(\overline{\Lambda}_{12}'+\overline{\Lambda}_{21}'\right)\right.\\ &+ \left.\overline{\Lambda}_{22}'\,k_2^2\right\}+\frac{(\,v_R^2+v_L^2)}{4}\left\{k_1k_1'\,\Omega_{11}+\Omega_{12}k_1\,k_2'+\Omega_{21}k_2\,k_1'+k_2k_2'\,\Omega_{22}\right\}\\ &+ \frac{(\,v_R^2+v_L^2)}{4}\left\{k_1k_1'\,\Omega_{11}'+\Omega_{12}'k_2\,k_1'+\Omega_{21}'k_1\,k_2'+k_2k_2'\,\Omega_{22}'\right\}\end{aligned}$$

(f) The minimum scalar potential $\langle V^{(4e)}\rangle$:

$$\langle V^{(4e)}\rangle = \frac{\lambda_{LR}}{4}\left(v_L^4+v_R^4\right)$$

## 3. The constrains equations:

Knowing the total scalar potential:

$$\langle V\rangle = \langle V^{(2)}\rangle+\langle V^{(4a)}\rangle+\langle V^{(4b)}\rangle+\langle V^{(4c)}\rangle+\langle V^{(4d)}\rangle+\langle V^{(4e)}\rangle,$$

the constrains equations are the followings:

- $\frac{\partial \langle V\rangle}{\partial k_1} = t_1;$

$$
\begin{aligned}
t_1 &= \mu_{11}^2\, k_1 + \left(\lambda_{11}+\lambda'_{11}\right) k_1^3 + k_1 k_1'^2 \left(\lambda'_{11}+\tilde\lambda_{11}+2\,\tilde\lambda'_{11}\right) + \tilde\mu_{11}^2\, k'_1 + \frac{k_1}{2}\Big(\overline{\Lambda}'_{11} v_R^2 + \Lambda_{11} v_R^2 + \\
&+ \tilde\lambda_{21} k_2'^2 + \tilde\lambda_{12} k_2'^2 + \tilde\lambda'_{21} k_2'^2 + \tilde\lambda'_{12} k_2'^2 + \left(\overline{\Lambda}'_{11}+\Lambda_{11}\right) v_L^2 + k_2^2\left(\lambda'_{12}+\lambda'_{21}+\lambda_{21}+\lambda_{12}+\rho_{12}\right)\Big) + \\
&+ \frac{k'_1 v_R^2}{4}\left(\Omega'_{11}+2\,\tilde\Lambda_{11}+\Omega_{11}\right) + \frac{k'_2 v_R^2}{4}\left(\Omega'_{21}+\Omega_{12}+\tilde\Lambda_{12}+\tilde\Lambda_{21}\right) + \frac{k_2 v_R^2}{4}\Big(\overline{\Lambda}'_{21}+\overline{\Lambda}'_{12} \\
&+ \Lambda_{12}+\Lambda_{21}) + \frac{k_2 k'_1 k'_2}{2}\left(\tilde\lambda'_{21}+\tilde\lambda'_{12}+\lambda'_{12}+\lambda'_{21}+\tilde\rho_{12}\right) + \frac{k'_1 v_L^2}{4}\left(\Omega'_{11}+\Omega_{11}+2\,\tilde\Lambda_{11}\right) \\
&+ \frac{k'_2 v_L^2}{4}\Big(\tilde\Lambda_{12}+\tilde\Lambda_{21}+\Omega'_{21}++\,\Omega_{12}) + \frac{k'_2}{2}\left(\tilde\mu_{12}^2+\tilde\mu_{21}^2\right) + \frac{k_2 v_L^2}{4}\left(\overline{\Lambda}'_{12}+\overline{\Lambda}'_{21}+\Lambda_{12}+\Lambda_{21}\right) \\
&+ \frac{k_2}{2}(\mu_{12}^2+\mu_{21}^2)
\end{aligned}
$$

- $\frac{\partial \langle V\rangle}{\partial k'_1} = t'_1;$

$$
\begin{aligned}
t'_1 &= \tilde\mu_{11}^2\, k_1 + \left(\lambda_{11}+\lambda'_{11}\right) k_1'^3 + k'_1 k_1^2 \left(\lambda'_{11}+\tilde\lambda_{11}+2\,\tilde\lambda'_{11}\right) + \mu_{11}^2\, k'_1 + \frac{k'_1}{2}\Big(\overline{\Lambda}_{11} v_R^2 + \Lambda_{11} v_R^2 + \\
&+ \tilde\lambda'_{12} k_2^2 + \tilde\lambda'_{21} k_2^2 + \tilde\lambda_{12} k_2^2 + \tilde\lambda_{21} k_2^2 + \left(\overline{\Lambda}_{11}+\Lambda_{11}\right) v_L^2 + k_2'^2\left(\lambda'_{12}+\lambda'_{21}+\lambda_{21}+\lambda_{12}+\rho_{12}\right)\Big) + \\
&+ \frac{k_1 v_R^2}{4}\left(\Omega'_{11}+2\,\tilde\Lambda_{11}+\Omega_{11}\right) + \frac{k_2 v_R^2}{4}\left(\Omega'_{12}+\Omega_{21}+\tilde\Lambda_{21}+\tilde\Lambda_{12}\right) + \frac{k'_2 v_R^2}{4}\Big(\overline{\Lambda}_{21}+\overline{\Lambda}_{12} \\
&+ \Lambda_{12}+\Lambda_{21}) + \frac{k_2 k_1 k'_2}{2}\left(\tilde\lambda'_{12}+\tilde\lambda'_{21}+\lambda'_{12}+\lambda'_{21}+\tilde\rho_{12}\right) + \frac{k_1 v_L^2}{4}\left(\Omega'_{11}+\Omega_{11}+2\,\tilde\Lambda_{11}\right) \\
&+ \frac{k_2 v_L^2}{4}\left(\tilde\Lambda_{12}+\tilde\Lambda_{21}+\Omega'_{12}+\Omega_{21}\right) + \frac{k_2}{2}\left(\tilde\mu_{12}^2+\tilde\mu_{21}^2\right) + \frac{k'_2 v_L^2}{4}\left(\overline{\Lambda}_{12}+\overline{\Lambda}_{21}+\Lambda_{12}+\Lambda_{21}\right) \\
&+ \frac{k'_2}{2}(\mu_{12}^2+\mu_{21}^2)
\end{aligned}
$$

- $\frac{\partial \langle V\rangle}{\partial k_2} = t_2;$

$$
\begin{aligned}
t_2 &= \mu_{22}^2\, k_2 + \left(\lambda_{22}+\lambda'_{22}\right) k_2^3 + k_2 k_2'^2 \left(\lambda'_{22}+\tilde\lambda_{22}+2\,\tilde\lambda'_{22}\right) + \tilde\mu_{22}^2\, k'_2 + \frac{k_2}{2}\Big(\overline{\Lambda}'_{22} v_R^2 + \Lambda_{22} v_R^2 + \\
&+ \tilde\lambda_{21} k_1'^2 + \tilde\lambda_{12} k_1'^2 + \tilde\lambda'_{21} k_1'^2 + \tilde\lambda'_{12} k_1'^2 + \left(\overline{\Lambda}'_{22}+\Lambda_{22}\right) v_L^2 + k_1^2\left(\lambda'_{12}+\lambda'_{21}+\lambda_{21}+\lambda_{12}+\rho_{12}\right)\Big) \\
&+ \frac{k'_2 v_R^2}{4}\left(\Omega'_{22}+2\,\tilde\Lambda_{22}+\Omega_{22}\right) + \frac{k'_1 v_R^2}{4}\left(\Omega'_{12}+\Omega_{21}+\tilde\Lambda_{12}+\tilde\Lambda_{21}\right) + \frac{k_1 v_R^2}{4}\Big(\overline{\Lambda}'_{12}+\overline{\Lambda}'_{21} \\
&+ \Lambda_{12}+\Lambda_{21}) + \frac{k_1 k'_1 k'_2}{2}\left(\tilde\lambda'_{21}+\tilde\lambda'_{12}+\lambda'_{12}+\lambda'_{21}+\tilde\rho_{12}\right) + \frac{k'_2 v_L^2}{4}\left(\Omega'_{22}+\Omega_{22}+2\,\tilde\Lambda_{22}\right) \\
&+ \frac{k'_1 v_L^2}{4}\left(\tilde\Lambda_{12}+\tilde\Lambda_{21}+\Omega'_{12}+\Omega_{21}\right) + \frac{k'_1}{2}\left(\tilde\mu_{12}^2+\tilde\mu_{21}^2\right) + \frac{k_1 v_L^2}{4}\left(\overline{\Lambda}'_{12}+\overline{\Lambda}'_{21}+\Lambda_{12}+\Lambda_{21}\right) \\
&+ \frac{k_1}{2}(\mu_{12}^2+\mu_{21}^2)
\end{aligned}
$$

- $\frac{\partial\langle V\rangle}{\partial k_2'} = t_2'$;

$$
\begin{aligned}
t_2' &= \mu_{22}^2\, k_2' + \left(\lambda_{22} + \lambda_{22}'\right) k_2'^3 + k_2' k_2^2 \left(\lambda_{22}' + \tilde{\lambda}_{22} + 2\,\tilde{\lambda}_{22}'\right) + \tilde{\mu}_{22}^2\, k_2 + \frac{k_2'}{2}\left(\overline{\Lambda}_{22} v_R^2 + \Lambda_{22} v_R^2 + \right.\\
&+ \left.\tilde{\lambda}_{21} k_1^2 + \tilde{\lambda}_{12} k_1^2 + \tilde{\lambda}_{21}' k_1^2 + \tilde{\lambda}_{12}' k_1^2 + \left(\overline{\Lambda}_{22} + \Lambda_{22}\right) v_L^2 + k_1'^2 \left(\lambda_{12}' + \lambda_{21}' + \lambda_{21} + \lambda_{12} + \rho_{12}\right)\right) + \\
&+ \frac{k_2 v_R^2}{4}\left(\Omega_{22}' + 2\,\tilde{\Lambda}_{22} + \Omega_{22}\right) + \frac{k_1 v_R^2}{4}\left(\Omega_{21}' + \Omega_{12} + \tilde{\Lambda}_{12} + \tilde{\Lambda}_{21}\right) + \frac{k_1' v_R^2}{4}\left(\overline{\Lambda}_{12} + \overline{\Lambda}_{21}\right.\\
&+ \left.\Lambda_{12} + \Lambda_{21}\right) + \frac{k_1 k_1' k_2}{2}\left(\tilde{\lambda}_{21}' + \tilde{\lambda}_{12}' + \lambda_{12}' + \lambda_{21}' + \tilde{\rho}_{12}\right) + \frac{k_2 v_L^2}{4}\left(\Omega_{22}' + \Omega_{22} + 2\,\tilde{\Lambda}_{22}\right)\\
&+ \frac{k_1 v_L^2}{4}\left(\tilde{\Lambda}_{12} + \tilde{\Lambda}_{21} + \Omega_{21}' + \Omega_{12}\right) + \frac{k_1}{2}\left(\tilde{\mu}_{12}^2 + \tilde{\mu}_{21}^2\right) + \frac{k_1' v_L^2}{4}\left(\overline{\Lambda}_{12} + \overline{\Lambda}_{21} + \Lambda_{12} + \Lambda_{21}\right)\\
&+ \frac{k_1'}{2}(\mu_{12}^2 + \mu_{21}^2)
\end{aligned}
$$

- $\frac{\partial\langle V\rangle}{\partial v_L} = t_L$;

$$
\begin{aligned}
t_L &= \frac{v_L}{2}\left\{k_1'^2(\Lambda_{11} + \overline{\Lambda}_{11}) + k_1' k_2'(\Lambda_{12} + \Lambda_{21} + \overline{\Lambda}_{12} + \overline{\Lambda}_{21}) + k_1' k_2(\tilde{\Lambda}_{12} + \tilde{\Lambda}_{21} + \Omega_{12}' + \Omega_{21}) + \right.\\
&+ k_1 k_1'(\Omega_{11}' + 2\,\tilde{\Lambda}_{11} + \Omega_{11}) + k_2 k_2'(\Omega_{22}' + 2\,\tilde{\Lambda}_{22} + \Omega_{22}) + k_1 k_2'(\Omega_{21}' + \tilde{\Lambda}_{21} + \tilde{\Lambda}_{12} + \Omega_{12}) + \\
&+ k_1 k_2(\overline{\Lambda}_{21}' + \overline{\Lambda}_{12}' + \Lambda_{21} + \Lambda_{12}) + k_1^2(\Lambda_{11} + \overline{\Lambda}_{11}') + k_2^2(\Lambda_{22} + \overline{\Lambda}_{22}') + k_2'^2(\Lambda_{22} + \overline{\Lambda}_{22})\\
&+ \left. 2\,\mu_{LR}^2 + 2\,v_L^2 \lambda_{LR}\right\}.
\end{aligned}
$$

- $\frac{\partial\langle V\rangle}{\partial v_R} = t_R$;

$$
\begin{aligned}
t_R &= \frac{v_R}{2}\left\{k_1'^2(\Lambda_{11} + \overline{\Lambda}_{11}) + k_1' k_2'(\Lambda_{12} + \Lambda_{21} + \overline{\Lambda}_{12} + \overline{\Lambda}_{21}) + k_1' k_2(\tilde{\Lambda}_{12} + \tilde{\Lambda}_{21} + \Omega_{12}' + \Omega_{21}) + \right.\\
&+ k_1 k_1'(\Omega_{11}' + 2\,\tilde{\Lambda}_{11} + \Omega_{11}) + k_2 k_2'(\Omega_{22}' + 2\,\tilde{\Lambda}_{22} + \Omega_{22}) + k_1 k_2'(\Omega_{21}' + \tilde{\Lambda}_{21} + \tilde{\Lambda}_{12} + \Omega_{12}) + \\
&+ k_1 k_2(\overline{\Lambda}_{21}' + \overline{\Lambda}_{12}' + \Lambda_{21} + \Lambda_{12}) + k_1^2(\Lambda_{11} + \overline{\Lambda}_{11}') + k_2^2(\Lambda_{22} + \overline{\Lambda}_{22}') + k_2'^2(\Lambda_{22} + \overline{\Lambda}_{22})\\
&+ \left. 2\mu_{LR}^2 + 2\,v_R^2 \lambda_{LR}\right\}.
\end{aligned}
$$

These are the six equations that restrict the values of the parameters proposed by our model.

# Appendix G

# Coupling the leptons with the gauge bosons, in the case $v_L = 0$

### Physical States as a function of symmetries states:

We have regarding in this case $v_L = 0$

$$W_\mu^{3L} = a_1\, A_\mu + a_2\, Z_{1\mu} + a_3\, Z_{2\mu} \tag{G.1}$$
$$W_\mu^{3R} = b_1\, A_\mu + b_2\, Z_{1\mu} + b_3\, Z_{2\mu}$$
$$B_\mu = c_1\, A_\mu + c_2\, Z_{1\mu} + c_3\, Z_{2\mu} \tag{G.2}$$

where:

$$a_1 = \frac{g'}{\sqrt{g^2 + 2g'^2}} \tag{G.3}$$

$$a_2 = \sqrt{\frac{g^2 + g'^2}{g^2 + 2g'^2}} - \frac{\sqrt{g_p^2 + g^2}\sqrt{2g'^2 + g^2}g^6 A'^2}{\left(g^2 + g'^2\right)^4 v_R^4} \tag{G.4}$$

$$a_3 = \frac{A' g^3}{(g^2 + g'^2)\sqrt{g^2 + g'^2}\, v_R^2} + \frac{2g^5 g'^2 A'^2 \sqrt{g^2 + g'^2}}{(g^2 + g'^2)^4 v_R^4} \tag{G.5}$$

$$b_1 = \frac{g'}{\sqrt{g^2 + 2g'^2}} \tag{G.6}$$

$$b_2 = \frac{-g'^2}{\sqrt{g'^2 + g^2}\sqrt{2g'^2 + g^2}} + \frac{g^4 A' \sqrt{2g'^2 + g^2}}{\sqrt{g'^2 + g^2}(g^2 + g'^2)^2 v_R^2} \tag{G.7}$$

$$b_3 = \frac{-g}{\sqrt{g^2 + g'^2}} - \frac{g^3\, A' g'^2}{\sqrt{g^2 + g'^2}\,(g^2 + g'^2)^2 v_R^2} \tag{G.8}$$

$$c_1 = \frac{g}{\sqrt{g^2 + 2g'^2}} \tag{G.9}$$

$$c_2 = \frac{-g\, g'}{\sqrt{g^2 + g'^2}\,\sqrt{g^2 + 2\, g'^2}} - \frac{g^3\, A' g' \sqrt{g^2 + 2\, g'^2}}{(g^2 + g'^2)^2 \sqrt{g^2 + g'^2} v_R^2} \tag{G.10}$$

$$c_3 = \frac{g'}{\sqrt{g^2 + g'^2}} - \frac{g^4\, A' g'}{\sqrt{g^2 + g'^2}\,(g^2 + g'^2)^2 v_R^2} \tag{G.11}$$

Remember that: $A' \equiv k_1^2 + k_2^2 + k_1'^2 + k_2'^2$.

$$\mathcal{L}^{lep} = i \begin{pmatrix} \overline{\nu}_L & \overline{\ell}_L \end{pmatrix} \left( \gamma^\mu \partial_\mu + i \frac{g_L}{2} \gamma^\mu \overline{\tau} \cdot \overline{W}^L_\mu - i \frac{g^{'}}{2} \gamma^\mu B_\mu \right) \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$$
$$+ \; i \begin{pmatrix} \overline{\nu}_R & \overline{\ell}_R \end{pmatrix} \left( \gamma^\mu \partial_\mu + i \frac{g_R}{2} \gamma^\mu \overline{\tau} \cdot \overline{W}^R_\mu - i \frac{g^{'}}{2} \gamma^\mu B_\mu \right) \begin{pmatrix} \nu_R \\ \ell_R \end{pmatrix} \quad \text{(G.12)}$$

however:

$$\overline{\tau} \cdot \overline{W}^L_\mu = \begin{pmatrix} W^L_{3\mu} & W^L_{1\mu} - i\, W^L_{2\mu} \\ W^L_{1\mu} + i\, W^L_{2\mu} & -W^L_{3\mu} \end{pmatrix}, \quad \text{(G.13)}$$

$$\overline{\tau} \cdot \overline{W}^R_\mu = \begin{pmatrix} W^R_{3\mu} & W^R_{1\mu} - i\, W^R_{2\mu} \\ W^R_{1\mu} + i\, W^R_{2\mu} & -W^R_{3\mu} \end{pmatrix}, \quad \text{(G.14)}$$

replacing in the previous expression, G.12, we have:

$$\mathcal{L}^{lep} = i \begin{pmatrix} \overline{\nu}_L & \overline{\ell}_L \end{pmatrix} \begin{pmatrix} \partial\!\!\!/ + i\frac{g_L}{2}\gamma^\mu W^L_{3\mu} - i\frac{g^{'}}{2}\gamma^\mu B_\mu & i\frac{g_L}{2}\gamma^\mu \left(W^L_{1\mu} - i\, W^L_{2\mu}\right) \\ i\frac{g_L}{2}\gamma^\mu \left(W^L_{1\mu} + i\, W^L_{2\mu}\right) & \partial\!\!\!/ - i\frac{g_L}{2}\gamma^\mu W^L_{3\mu} - i\frac{g^{'}}{2}\gamma^\mu B_\mu \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$$
$$+ \; i \begin{pmatrix} \overline{\nu}_R & \overline{\ell}_R \end{pmatrix} \begin{pmatrix} \partial\!\!\!/ + i\frac{g_R}{2}\gamma^\mu W^R_{3\mu} - i\frac{g^{'}}{2}\gamma^\mu B_\mu & i\frac{g_R}{2}\gamma^\mu \left(W^R_{1\mu} - i\, W^R_{2\mu}\right) \\ i\frac{g_R}{2}\gamma^\mu \left(W^R_{1\mu} + i\, W^R_{2\mu}\right) & \partial\!\!\!/ - i\frac{g_R}{2}\gamma^\mu W^R_{3\mu} - i\frac{g^{'}}{2}\gamma^\mu B_\mu \end{pmatrix} \begin{pmatrix} \nu_R \\ \ell_R \end{pmatrix}$$

multiplying the matrices and regarding the neutrinos contribution:

$$\begin{aligned} \mathcal{L}^{lep}_{\nu_\ell} &= i\,\overline{\nu}_L \left( \partial\!\!\!/ + i\frac{g_L}{2}\gamma^\mu W^L_{3\mu} - i\frac{g^{'}}{2}\gamma^\mu B_\mu \right) \nu_L + \overline{\nu}_R \left( \partial\!\!\!/ + i\frac{g_R}{2}\gamma^\mu W^R_{3\mu} - i\frac{g^{'}}{2}\gamma^\mu B_\mu \right) \nu_R \\ &= i\,\overline{\nu}_L\; \partial\!\!\!/ \nu_L + i\; \underbrace{\overline{\nu}_L \left( i\frac{g_L}{2}\gamma^\mu W^L_{3\mu} - i\frac{g^{'}}{2}\gamma^\mu B_\mu \right) \nu_L}_{(I)} + i\,\overline{\nu}_R\; \partial\!\!\!/ \nu_R + \\ &+ \; \underbrace{i\; \overline{\nu}_R \left( i\frac{g_R}{2}\gamma^\mu W^R_{3\mu} - i\frac{g^{'}}{2}\gamma^\mu B_\mu \right) \nu_R}_{(II)} \end{aligned}$$

1. **Coupling with the Electromagnetic field:**
   We know from the expression G.1:

$$\begin{aligned} W^{3L}_\mu &= \left( \frac{g^{'}}{\sqrt{g^2 + 2g^{'2}}} \right) A_\mu + \cdots \\ W^{3R}_\mu &= \left( \frac{g^{'}}{\sqrt{g^2 + 2g^{'2}}} \right) A_\mu + \cdots \\ B_\mu &= \left( \frac{g}{\sqrt{g^2 + 2g^{'2}}} \right) A_\mu + \cdots \end{aligned} \quad \text{(G.15)}$$

   working with $(I)$ and $(II)$:

$$
\begin{aligned}
(I)+(II) &= \frac{i}{2}\overline{\nu}_L\gamma^\mu \left( g \underbrace{W^L_{3\mu}}_{A_\mu \frac{g'}{\sqrt{g^2+2g'^2}}} - g' \underbrace{B_\mu}_{A_\mu \frac{g}{\sqrt{g^2+2g'^2}}} \right) \nu_L \\
&+ \frac{i}{2}\overline{\nu}_R\gamma^\mu \left( g \underbrace{W^R_{3\mu}}_{A_\mu \frac{g'}{\sqrt{g^2+2g'^2}}} - g' \underbrace{B_\mu}_{A_\mu \frac{g}{\sqrt{g^2+2g'^2}}} \right) \nu_R
\end{aligned} \tag{G.16}
$$

Where it has been necessary to do $g_L = g_R = g$, then we have:

$$
\begin{aligned}
(I)+(II) &\stackrel{A_\mu}{=} \frac{i}{2}\underbrace{\overline{\nu}_L}_{\overline{\nu_\ell}P_R}\gamma^\mu A_\mu \left( g\frac{g'}{\sqrt{g^2+2g'^2}} - g'\frac{g}{\sqrt{g^2+2g'^2}} \right) \underbrace{\nu_L}_{P_L\nu_\ell} + \\
&+ \frac{i}{2}\underbrace{\overline{\nu}_R}_{\overline{\nu}_\ell\, PL}\gamma^\mu A_\mu \left( g\frac{g'}{\sqrt{g^2+2g'^2}} - g'\frac{g}{\sqrt{g^2+2g'^2}} \right) \underbrace{\nu_R}_{P_R\,\nu_\ell} \\
&= \frac{i}{2}\overline{\nu_\ell}P_R\gamma^\mu A_\mu \left( g\frac{g'}{\sqrt{g^2+2g'^2}} - g'\frac{g}{\sqrt{g^2+2g'^2}} \right) P_L\nu_\ell + \\
&+ \frac{i}{2}\overline{\nu}_\ell P_L\gamma^\mu A_\mu \left( g\frac{g'}{\sqrt{g^2+2g'^2}} - g'\frac{g}{\sqrt{g^2+2g'^2}} \right) P_R\nu_\ell \\
&= \frac{i}{2}\overline{\nu_\ell}\gamma^\mu A_\mu \underbrace{\left( g\frac{g'}{\sqrt{g^2+2g'^2}} - g'\frac{g}{\sqrt{g^2+2g'^2}} \right)}_{=0} \nu_\ell \;=\; 0.
\end{aligned}
$$

This proves that the neutrino has zero electric charge.

In the same way for the case of leptons:

$$
\begin{aligned}
\mathcal{L}^{lep}_{\ell_\alpha} &= i\,\overline{\ell}_L \left( \partial\!\!\!/ - i\frac{g}{2}\gamma^\mu W^L_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu \right) \ell_L + i\,\overline{\ell}_R \left( \partial\!\!\!/ - i\frac{g}{2}\gamma^\mu W^R_{3\mu} - i\frac{g'}{2}\gamma^\mu B_\mu \right) \ell_R \\
&= i\,\overline{\ell}_L\,\partial\!\!\!/\ell_L + i\,\underbrace{(-1)\overline{\ell}_L \left( i\frac{g}{2}\gamma^\mu W^L_{3\mu} + i\frac{g'}{2}\gamma^\mu B_\mu \right) \ell_L}_{(I')} + i\,\overline{\ell}_R\,\partial\!\!\!/\ell_R + \\
&+ \;\; i\,\underbrace{(-1)\overline{\ell}_R \left( i\frac{g}{2}\gamma^\mu W^R_{3\mu} + i\frac{g'}{2}\gamma^\mu B_\mu \right) \ell_R}_{(II')}
\end{aligned}
$$

considering the interaction with the photon:

$$
\begin{aligned}
(I') + (II') &\overset{A_\mu}{=} \frac{-i}{2} \underbrace{\bar{\ell}_L}_{\bar{\ell} P_R} \gamma^\mu A_\mu \left( g \frac{g'}{\sqrt{g^2+2g'^2}} + g' \frac{g}{\sqrt{g^2+2g'^2}} \right) \underbrace{\ell_L}_{P_L \ell} \\
&+ \frac{-i}{2} \underbrace{\bar{\ell}_R}_{\bar{\ell}\, PL} \gamma^\mu A_\mu \left( g \frac{g'}{\sqrt{g^2+2g'^2}} + g' \frac{g}{\sqrt{g^2+2g'^2}} \right) \underbrace{\ell_R}_{P_R\, \ell} \\
&= \frac{-i}{2} \bar{\ell} P_R \gamma^\mu A_\mu \left( g \frac{g'}{\sqrt{g^2+2g'^2}} + g' \frac{g}{\sqrt{g^2+2g'^2}} \right) P_L \ell \\
&+ \frac{-i}{2} \bar{\ell}_\ell P_L \gamma^\mu A_\mu \left( g \frac{g'}{\sqrt{g^2+2g'^2}} + g' \frac{g}{\sqrt{g^2+2g'^2}} \right) P_R \ell \\
&= \frac{-i}{2} \bar{\ell} \gamma^\mu A_\mu \underbrace{\left( g \frac{g'}{\sqrt{g^2+2g'^2}} + g' \frac{g}{\sqrt{g^2+2g'^2}} \right)}_{=2\frac{g' g}{\sqrt{g^2+2g'^2}}} \ell.
\end{aligned}
$$

therefore, we have the electric charge for charged leptons:

$$
e = \frac{-g' g}{\sqrt{g^2+2g'^2}}
$$

2. **Coupling with $Z_1$ (Neutral current):**

$$
\begin{aligned}
\mathcal{L}_{\nu_\alpha} &= -\bar{\nu}_L \left( \frac{g}{2} \gamma^\mu \underbrace{W^L_{3\mu}}_{a_2\, Z_{1\mu}} - \frac{g'}{2} \gamma^\mu \underbrace{B_\mu}_{c_2\, Z_{1\mu}} \right) \nu_L - \bar{\nu}_R \left( \frac{g}{2} \gamma^\mu \underbrace{W^R_{3\mu}}_{b_2\, Z_{1\mu}} - \frac{g'}{2} \gamma^\mu \underbrace{B_\mu}_{c_2\, Z_{1\mu}} \right) \nu_R \\
&= -\frac{1}{2} \bar{\nu}_L \gamma^\mu \left( a_2\, g - c_2\, g' \right) Z_{1\mu}\, \nu_L - \frac{1}{2} \bar{\nu}_R \gamma^\mu \left( b_2\, g - c_2\, g' \right) Z_{1\mu}\, \nu_R \\
&= -\frac{1}{2} \left[ \bar{\nu}\, P_R \gamma^\mu \left( a_2\, g - c_2\, g' \right) Z_{1\mu} P_L \nu + \bar{\nu}\, P_L \gamma^\mu \left( b_2\, g - c_2\, g' \right) Z_{1\mu} P_R \nu \right] \\
&= -\frac{1}{2} \left[ \bar{\nu}\, \gamma^\mu\, P_L \left( a_2\, g - c_2\, g' \right) Z_{1\mu} P_L \nu + \bar{\nu}\, \gamma^\mu\, P_R \left( b_2\, g - c_2\, g' \right) Z_{1\mu} P_R \nu \right] \\
&= -\frac{1}{2} \left[ \bar{\nu}\, \gamma^\mu\, P_L^2 \left( a_2\, g - c_2\, g' \right) Z_{1\mu} \nu + \bar{\nu}\, \gamma^\mu\, P_R^2 \left( b_2\, g - c_2\, g' \right) Z_{1\mu} \nu \right] \\
&= -\frac{1}{2} \left[ \bar{\nu}\, \gamma^\mu\, P_L \underbrace{\left( a_2\, g - c_2\, g' \right)}_{=a_L^{\nu_\ell}} Z_{1\mu} \nu + \bar{\nu}\, \gamma^\mu\, P_R \underbrace{\left( b_2\, g - c_2\, g' \right)}_{=b_R^{\nu_\ell}} Z_{1\mu} \nu \right] \\
&= -\frac{1}{2} \bar{\nu}\, \gamma^\mu \left[ \frac{1}{2}(1-\gamma_5)\, a_L^{\nu_\ell} + \frac{1}{2}(1+\gamma_5)\, b_R^{\nu_\ell} \right] Z_{1\mu}\, \nu \\
&= -\frac{1}{2} \bar{\nu}\, \gamma^\mu \left[ \frac{a_L^{\nu_\ell} + b_R^{\nu_\ell}}{2} - \frac{a_L^{\nu_\ell} - b_R^{\nu_\ell}}{2} \gamma_5 \right] Z_{1\mu}\, \nu = -\frac{g}{2\cos\theta} \bar{\nu}\, \gamma^\mu \left( g_V^{\nu_\ell} - g_A^{\nu_\ell} \gamma_5 \right) Z_{1\mu} \nu.
\end{aligned}
$$

where:

$$
\begin{aligned}
a_L^{\nu_\ell} &= g\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}} + \frac{A'\,g'^2 g^3}{v_R^2(g^2+g'^2)^2}\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}} && \text{(G.17)}\\
b_R^{\nu_\ell} &= \frac{A'\,g^3}{(g^2+g'^2)\,v_R^2}\sqrt{\frac{g^2+2g'^2}{g^2+\,g'^2}} && \text{(G.18)}\\
\frac{g}{\cos\theta}g_V^{\nu} &= \frac{1}{2}\,(a_L^{\nu_\ell}+b_R^{\nu_\ell}) && \text{(G.19)}\\
\frac{g}{\cos\theta}g_A^{\nu} &= \frac{1}{2}\,(a_L^{\nu_\ell}-b_R^{\nu_\ell}) && \text{(G.20)}
\end{aligned}
$$

remember that:

$$
g\sin\theta \;=\; g'\sqrt{\cos 2\theta}\;\rightarrow\;\sqrt{\frac{g^2+g'^2}{g^2+2\,g'^2}} = \cos\theta
$$

Finally:

$$
\begin{aligned}
a_L^{\nu_\ell} &= \frac{g}{\cos\theta}\left[1+\frac{A'\,g'^2 g^2}{v_R^2(g^2+g'^2)^2}\right] && \text{(G.21)}\\
b_R^{\nu_\ell} &= \frac{A'\,g^3}{(g^2+g'^2)\cos\theta\,v_R^2} && \text{(G.22)}\\
g_V^{\nu_\ell} &= \frac{1}{2}+\frac{A'\,g'^2 g^2}{2\,v_R^2(g^2+g'^2)^2}+\frac{A'\,g^2}{2(g^2+g'^2)\,v_R^2} && \text{(G.23)}\\
g_A^{\nu_\ell} &= \frac{1}{2}+\frac{A'\,g'^2 g^2}{2\,v_R^2(g^2+g'^2)^2}-\frac{A'\,g^2}{2(g^2+g'^2)\,v_R^2} && \text{(G.24)}
\end{aligned}
$$

(a) Summarizing we have for the case of neutrinos:

$$
\begin{aligned}
g_V^{\nu_\ell} &= \frac{1}{2}+\frac{A'\,g'^2 g^2}{2\,v_R^2(g^2+g'^2)^2}+\frac{A'\,g^2}{2(g^2+g'^2)\,v_R^2}\\
g_A^{\nu_\ell} &= \frac{1}{2}+\frac{A'\,g'^2 g^2}{2\,v_R^2(g^2+g'^2)^2}-\frac{A'\,g^2}{2(g^2+g'^2)\,v_R^2}
\end{aligned}
\qquad \text{(G.25)}
$$

but we know the following:

$$
\frac{g}{\sqrt{g^2+2\,g'^2}} = \sqrt{\cos 2\theta}, \qquad\qquad \frac{g\,g'}{g^2+g'^2} = \frac{\sin\theta\,\sqrt{\cos 2\theta}}{\cos^2\theta}
$$

replacing in G.25, we have:

$$
\begin{aligned}
g_V^{\nu_\ell} &= \frac{1}{2}+\frac{A'\,\sin^2\theta\,\cos 2\theta}{2\,v_R^2\,\cos^4\theta}+\frac{A'\,\cos 2\theta}{2\,v_R^2\cos^2\theta}\\
g_A^{\nu_\ell} &= \frac{1}{2}+\frac{A'\,\sin^2\theta\,\cos 2\theta}{2\,v_R^2\,\cos^4\theta}-\frac{A'\,\cos 2\theta}{2\,v_R^2\cos^2\theta}
\end{aligned}
\qquad \text{(G.26)}
$$

(b) for the case of charged leptons:

$$
\begin{aligned}
\mathcal{L}_{\ell_\alpha} &= \overline{\ell}_L \left( \frac{g}{2}\gamma^\mu \underbrace{W^L_{3\mu}}_{a_2\, Z_{1\mu}} + \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_2\, Z_{1\mu}} \right) \ell_L + \overline{\ell}_R \left( \frac{g}{2}\gamma^\mu \underbrace{W^R_{3\mu}}_{b_2\, Z_{1\mu}} + \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_2\, Z_{1\mu}} \right) \ell_R \\
&= \frac{1}{2}\overline{\ell}_L\gamma^\mu \left(a_2\, g + c_2\, g'\right)\, Z_{1\mu}\, \ell_L + \frac{1}{2}\overline{\ell}_R\gamma^\mu \left(b_2\, g + c_2\, g'\right)\, Z_{1\mu}\, \ell_R \\
&= \frac{1}{2}\left[\overline{\ell}\, P_R\gamma^\mu \left(a_2\, g + c_2\, g'\right) Z_{1\mu} P_L \ell + \overline{\ell}\, P_L\gamma^\mu \left(b_2\, g + c_2\, g'\right) Z_{1\mu} P_R \ell\right] \\
&= \frac{1}{2}\left[\overline{\ell}\,\gamma^\mu\, P_L \left(a_2\, g + c_2\, g'\right) Z_{1\mu} P_L \ell + \overline{\ell}\,\gamma^\mu\, P_R \left(b_2\, g + c_2\, g'\right) Z_{1\mu} P_R \ell\right] \\
&= \frac{1}{2}\left[\overline{\ell}\,\gamma^\mu\, P_L^2 \left(a_2\, g + c_2\, g'\right) Z_{1\mu} \ell + \overline{\ell}\,\gamma^\mu\, P_R^2 \left(b_2\, g + c_2\, g'\right) Z_{1\mu} \ell\right] \\
&= \frac{1}{2}\left[\overline{\ell}\,\gamma^\mu\, P_L \underbrace{\left(a_2\, g + c_2\, g'\right)}_{=a^\ell_L} Z_{1\mu} \ell + \overline{\ell}\,\gamma^\mu\, P_R \underbrace{\left(b_2\, g + c_2\, g'\right)}_{=b^\ell_R} Z_{1\mu} \ell\right] \\
&= \frac{1}{2}\overline{\ell}\,\gamma^\mu \left[\frac{1}{2}(1-\gamma_5)\, a^\ell_L + \frac{1}{2}(1+\gamma_5)\, b^\ell_R\right] Z_{1\mu}\, \ell \\
&= \frac{1}{2}\overline{\ell}\,\gamma^\mu \left[\frac{a^\ell_L + b^\ell_R}{2} - \frac{a^\ell_L - b^\ell_R}{2}\gamma_5\right] Z_{1\mu}\, \ell = \frac{g}{2\cos\theta}\overline{\ell}\,\gamma^\mu \left(g_V^{\ell_\alpha} - g_A^{\ell_\alpha}\,\gamma_5\right) Z_{1\mu}\ell.
\end{aligned}
$$

where:

$$a_L^{\ell_\alpha} = \frac{g^3}{g^2+2g'^2}\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}} - \frac{A'\, g'^2 g^3}{v_R^2(g^2+g'^2)^2}\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}} \tag{G.27}$$

$$b_R^{\ell_\alpha} = -\frac{2g\, g'^2}{g^2+2g'^2}\sqrt{\frac{g^2+2g'^2}{g^2+g'^2}} \tag{G.28}$$

$$\frac{g}{\cos\theta} g_V^{\ell_\alpha} = \frac{1}{2}\left(a_L^{\ell_\alpha} + b_R^{\ell_\alpha}\right) \tag{G.29}$$

$$\frac{g}{\cos\theta} g_A^{\ell_\alpha} = \frac{1}{2}\left(a_L^{\ell_\alpha} - b_R^{\ell_\alpha}\right) \tag{G.30}$$

Knowing that:

$$g\sin\theta = g'\sqrt{\cos 2\theta} \;\rightarrow\; \sqrt{\frac{g^2+g'^2}{g^2+2\,g'^2}} = \cos\theta,$$

we have:

$$
\begin{aligned}
a_L^{\ell_\alpha} &= \frac{g}{\cos\theta}\left[\frac{g^2}{g^2+2g'^2} - \frac{A'\, g'^2 g^2}{v_R^2(g^2+g'^2)^2}\right] \\
b_R^{\ell_\alpha} &= \frac{-2\, g\, g'^2}{(g^2+2g'^2)\cos\theta},
\end{aligned} \tag{G.31}
$$

also:

$$
\begin{aligned}
g_V^{\ell_\alpha} &= \frac{1}{2}\left(\frac{g^2-2g'^2}{g^2+2g'^2}\right) - \frac{A'\, g'^2 g^2}{2\, v_R^2(g^2+g'^2)^2} \\
g_A^{\ell_\alpha} &= \frac{1}{2} - \frac{A'\, g'^2 g^2}{2\, v_R^2(g^2+g'^2)^2},
\end{aligned} \tag{G.32}
$$

taking into account the following:

$$\frac{g^2 - 2g'^2}{g^2 + 2g'^2} = 2\cos 2\theta - 1, \tag{G.33}$$

$$\frac{g\,g'}{g^2 + g'^2} = \frac{\sin\theta\sqrt{\cos 2\theta}}{\cos^2\theta}, \tag{G.34}$$

replacing in G.32, we finally have:

$$\begin{aligned} g_V^{\ell_\alpha} &= \frac{1}{2}\left(2\cos 2\theta - 1\right) - \frac{A'\,\sin^2\theta\,\cos 2\theta}{2v_R^2\cos^4\theta}, \\ g_A^{\ell_\alpha} &= \frac{1}{2} - \frac{A'\,\sin^2\theta\,\cos 2\theta}{2v_R^2\cos^4\theta}. \end{aligned} \tag{G.35}$$

3. **Coupling with $Z_2$ (Neutral current):**

$$\begin{aligned} \mathcal{L}_{\nu_\alpha} &= -\overline{\nu}_L\left(\frac{g}{2}\gamma^\mu \underbrace{W_{3\mu}^L}_{a_3 Z_{2\mu}} - \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_3 Z_{2\mu}}\right)\nu_L - \overline{\nu}_R\left(\frac{g}{2}\gamma^\mu \underbrace{W_{3\mu}^R}_{b_3 Z_{2\mu}} - \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_3 Z_{2\mu}}\right)\nu_R \\ &= -\frac{1}{2}\overline{\nu}_L\gamma^\mu\left(a_3\,g - c_3\,g'\right)\,Z_{2\mu}\,\nu_L - \frac{1}{2}\overline{\nu}_R\gamma^\mu\left(b_3\,g - c_3\,g'\right)\,Z_{2\mu}\,\nu_R \\ &= -\frac{1}{2}\left[\overline{\nu}\,P_R\gamma^\mu\left(a_3\,g - c_3\,g'\right)Z_{2\mu}P_L\nu + \overline{\nu}\,P_L\gamma^\mu\left(b_3\,g - c_3\,g'\right)Z_{2\mu}P_R\nu\right] \\ &= -\frac{1}{2}\left[\overline{\nu}\,\gamma^\mu\,P_L\left(a_3\,g - c_3\,g'\right)Z_{2\mu}P_L\nu + \overline{\nu}\,\gamma^\mu\,P_R\left(b_3\,g - c_3\,g'\right)Z_{2\mu}P_R\nu\right] \\ &= -\frac{1}{2}\left[\overline{\nu}\,\gamma^\mu\,P_L^2\left(a_3\,g - c_3\,g'\right)Z_{2\mu}\nu + \overline{\nu}\,\gamma^\mu\,P_R^2\left(b_3\,g - c_3\,g'\right)Z_{2\mu}\nu\right] \\ &= -\frac{1}{2}\left[\overline{\nu}\,\gamma^\mu\,P_L\underbrace{\left(a_3\,g - c_3\,g'\right)}_{=a_L'^{\nu_\ell}}Z_{2\mu}\nu + \overline{\nu}\,\gamma^\mu\,P_R\underbrace{\left(b_3\,g - c_3\,g'\right)}_{=b_R'^{\nu_\ell}}Z_{1\mu}\nu\right] \\ &= -\frac{1}{2}\overline{\nu}\,\gamma^\mu\left[\frac{1}{2}(1-\gamma_5)\,a_L'^{\nu_\ell} + \frac{1}{2}(1+\gamma_5)\,b_R'^{\nu_\ell}\right]Z_{2\mu}\,\nu \\ &= -\frac{1}{2}\overline{\nu}\,\gamma^\mu\left[\frac{a_L'^{\nu_\ell} + b_R'^{\nu_\ell}}{2} - \frac{a_L'^{\nu_\ell} - b_R'^{\nu_\ell}}{2}\gamma_5\right]Z_{2\mu}\,\nu = -\frac{g}{2\cos\theta\sqrt{\cos 2\theta}}\overline{\nu}\,\gamma^\mu\left(g_V'^{\nu_\ell} - g_A'^{\nu_\ell}\,\gamma_5\right)Z_{2\mu}\nu. \end{aligned}$$

In the same way of the previous section:

$$a_L'^{\nu_\ell} = -\frac{g'^2}{\sqrt{g^2 + g'^2}} + \frac{g^4\,A'\sqrt{g^2 + g'^2}(g^2 + 2g'^2)}{(g^2 + g'^2)^3 v_R^2} \tag{G.36}$$

$$b_R'^{\nu_\ell} = -\sqrt{g^2 + g'^2} + \frac{g^6\,A'^2\sqrt{g^2 + g'^2}(g^2 + 2g'^2)}{2(g^2 + g'^2)^4 v_R^4} \tag{G.37}$$

$$g_V'^{\nu_\ell} = \frac{1}{2} - \frac{A'\,\cos^2 2\theta}{2\,\cos^4\theta\,v_R^2} - \frac{A'^2\,\cos^3 2\theta}{4\,v_R^4\cos^6\theta} \tag{G.38}$$

$$g_A'^{\nu_\ell} = -\frac{\cos 2\theta}{2} - \frac{A'\,\cos^2 2\theta}{2\,\cos^4\theta\,v_R^2} + \frac{A'^2\,\cos^3 2\theta}{4\,v_R^4\cos^6\theta} \tag{G.39}$$

(a) Summarizing we have for the case of neutrinos:

$$
\begin{aligned}
g_V^{\prime\nu_\ell} &= \frac{1}{2} - \frac{A' \cos^2 2\theta}{2\,\cos^4\theta\, v_R^2} - \frac{A'^2\,\cos^3 2\theta}{4\,v_R^4 \cos^6\theta} \\
g_A^{\prime\nu_\ell} &= -\frac{\cos 2\theta}{2} - \frac{A' \cos^2 2\theta}{2\,\cos^4\theta\, v_R^2} + \frac{A'^2\,\cos^3 2\theta}{4\,v_R^4 \cos^6\theta}.
\end{aligned} \tag{G.40}
$$

(b) for the case of charged leptons:

$$
\begin{aligned}
\mathcal{L}_{\ell_\alpha} &= \bar{\ell}_L \left( \frac{g}{2}\gamma^\mu \underbrace{W^L_{3\mu}}_{a_3\,Z_{2\mu}} + \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_3\,Z_{2\mu}} \right) \ell_L + \bar{\ell}_R \left( \frac{g}{2}\gamma^\mu \underbrace{W^R_{3\mu}}_{b_3\,Z_{2\mu}} + \frac{g'}{2}\gamma^\mu \underbrace{B_\mu}_{c_3\,Z_{2\mu}} \right) \ell_R \\
&= \frac{1}{2}\bar{\ell}_L\gamma^\mu \left(a_3\,g + c_3\,g'\right)\, Z_{2\mu}\,\ell_L + \frac{1}{2}\bar{\ell}_R\gamma^\mu \left(b_3\,g + c_3\,g'\right)\, Z_{2\mu}\,\ell_R \\
&= \frac{1}{2}\left[\bar{\ell}\,P_R\gamma^\mu \left(a_3\,g + c_3\,g'\right) Z_{2\mu}P_L\ell + \bar{\ell}\,P_L\gamma^\mu \left(b_3\,g + c_3\,g'\right) Z_{2\mu}P_R\ell\right] \\
&= \frac{1}{2}\left[\bar{\ell}\,\gamma^\mu\,P_L \left(a_3\,g + c_3\,g'\right) Z_{2\mu}P_L\ell + \bar{\ell}\,\gamma^\mu\,P_R \left(b_3\,g + c_3\,g'\right) Z_{2\mu}P_R\ell\right] \\
&= \frac{1}{2}\left[\bar{\ell}\,\gamma^\mu\,P_L^2 \left(a_3\,g + c_3\,g'\right) Z_{2\mu}\ell + \bar{\ell}\,\gamma^\mu\,P_R^2 \left(b_3\,g + c_3\,g'\right) Z_{2\mu}\ell\right] \\
&= \frac{1}{2}\left[\bar{\ell}\,\gamma^\mu\,P_L \underbrace{\left(a_3\,g + c_3\,g'\right)}_{=a_L'^\ell} Z_{2\mu}\ell + \bar{\ell}\,\gamma^\mu\,P_R \underbrace{\left(b_3\,g + c_3\,g'\right)}_{=b_R'^\ell} Z_{2\mu}\ell\right] \\
&= \frac{1}{2}\bar{\ell}\,\gamma^\mu \left[\frac{1}{2}(1-\gamma_5)\,a_L'^\ell + \frac{1}{2}(1+\gamma_5)\,b_R'^\ell\right] Z_{2\mu}\,\ell \\
&= \frac{1}{2}\bar{\ell}\,\gamma^\mu \left[\frac{a_L'^\ell + b_R'^\ell}{2} - \frac{a_L'^\ell - b_R'^\ell}{2}\gamma_5\right] Z_{2\mu}\,\ell = \frac{g}{2\cos\theta\sqrt{\cos 2\theta}}\bar{\ell}\,\gamma^\mu\left(g_V'^{\ell_\alpha} - g_A'^{\ell_\alpha}\,\gamma_5\right) Z_{2\mu}\ell.
\end{aligned}
$$

where:

$$
a_L'^{\ell_\alpha} = \frac{g'^2}{\sqrt{g^2+g'^2}} + \frac{A' g^6\sqrt{g^2+g'^2}}{(g^2+g'^2)^3 v_R^2} \tag{G.41}
$$

$$
b_R'^{\ell_\alpha} = -\frac{(g^2-g'^2)\sqrt{g^2+g'^2}}{g^2+g'^2} - \frac{2A' g'^2 g^4\sqrt{g^2+g'^2}}{(g^2+g'^2)^3 v_R^2} \tag{G.42}
$$

$$
\frac{g}{\cos\theta\,\sqrt{\cos 2\theta}}\,g_V'^{\ell_\alpha} = \frac{1}{2}\left(a_L'^{\ell_\alpha} + b_R'^{\ell_\alpha}\right) \tag{G.43}
$$

$$
\frac{g}{\cos\theta\sqrt{\cos 2\theta}}\,g_A'^{\ell_\alpha} = \frac{1}{2}\left(a_L'^{\ell_\alpha} - b_R'^{\ell_\alpha}\right) \tag{G.44}
$$

Knowing that:

$$
g\sin\theta = g'\sqrt{\cos 2\theta}
$$

we will have:

$$
\begin{aligned}
a_L'^{\ell_\alpha} &= \frac{g}{\cos\theta\,\sqrt{\cos 2\theta}}\left(\sin^2\theta + \frac{A'\cos^3 2\theta}{v_R^2\,\cos^4\theta}\right) \\
b_R'^{\ell_\alpha} &= \frac{g}{\cos\theta\,\sqrt{\cos 2\theta}}\left(3\,\sin^2\theta - 1 - \frac{2A'\sin^2\theta\cos^2 2\theta}{v_R^2\cos^4\theta}\right)
\end{aligned} \tag{G.45}
$$

and also:

$$
\begin{aligned}
g_V^{\prime\ell_\alpha} &= \frac{1}{2}(4\sin^2\theta - 1) + \frac{A'\cos^3 2\theta}{2v_R^2\cos^4\theta} - \frac{A'\sin^2\theta\cos^2 2\theta}{v_R^2\cos^4\theta} \\
g_A^{\prime\ell_\alpha} &= \frac{\cos 2\theta}{2} + \frac{1}{2}\sin^2\theta\,(-3 + cos\theta) + \frac{A'\cos^3 2\theta}{2v_R^2\cos^4\theta} + \frac{A'\sin^2\theta\cos^2 2\theta}{v_R^2\cos^4\theta}.
\end{aligned}
\tag{G.46}
$$

# Appendix H

# Assignment of electrical charge to scalar multiplets

It will be considered one of the bi-doublets, $\Phi_1$, which gives mass to the charged leptons and neutrinos, in which we will demonstrate why their elements have the charge they have. Similarly it can be shown for the second bi-doublet, $\Phi_2$, which is responsible for giving mass to the quarks and their interactions.

As for the electric charge we know from the fermion sector, that it has the general form:

$$Q = T_{3L} + T_{3R} + \frac{1}{2}(B - L), \tag{H.1}$$

for all multiplets where $T_{3L}$ and $T_{3R}$ are generators of $SU(2)_L$ and $SU(2)_R$, respectively. This allows to calculate the charge of the fields in:

$$\Phi_1 = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}, \tag{H.2}$$

using:

$$Q\Phi_1 = \left[\frac{1}{2}\tau_3, \Phi_1\right] + \frac{1}{2}(B - L)\Phi_1, \tag{H.3}$$

however, $\Phi_1$ has a zero value of hypercharge, that is, $B - L = 0$, hence, the previous expression has the form:

$$Q\Phi_1 = \left[\frac{1}{2}\tau_3, \Phi_1\right] = \frac{1}{2}\left[\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}}_{=\begin{pmatrix} \phi_{11} & \phi_{12} \\ -\phi_{21} & -\phi_{22} \end{pmatrix}} - \underbrace{\begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{=\begin{pmatrix} \phi_{11} & -\phi_{12} \\ \phi_{21} & -\phi_{22} \end{pmatrix}} = \right] \tag{H.4}$$

then, we have:

$$Q\Phi_1 = \frac{1}{2}\begin{pmatrix} 0\phi_{11} & 2\phi_{12} \\ -2\phi_{21} & 0\phi_{22} \end{pmatrix} = \begin{pmatrix} (0)\phi_{11} & (+1)\phi_{12} \\ (-1)\phi_{21} & (0)\phi_{22} \end{pmatrix}, \tag{H.5}$$

where the numbers in parentheses represent the electric charges of the fields within the bi-doublet.

The bi-doublets can be expressed in the fundamental representation of the group $SU(2)$, simply expressed as a linear combination of those generated from the group $SU(2)$, which are the matrices of pauli: $\tau_1$, $\tau_2$ and $\tau_3$.

$$\begin{aligned}
\Phi_1 &= \frac{1}{2}\tau^i\,\delta^i = \frac{1}{2}\left(\tau^1\delta^1 + \tau^2\delta^2 + \tau^3\delta^3\right) \\
&= \frac{1}{2}\left\{\delta^1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \delta^2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \delta^3\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\} \\
\Phi_1 &= \frac{1}{2}\begin{pmatrix} \delta^3 & \delta^1 - i\,\delta^2 \\ \delta^1 + i\,\delta^2 & -\delta^3 \end{pmatrix} = \begin{pmatrix} \delta^3/2 & \frac{\delta^1 - i\,\delta^2}{2} \\ \frac{\delta^1 + i\,\delta^2}{2} & -\delta^3/2 \end{pmatrix} \qquad \text{(H.6)}
\end{aligned}$$

Finally, using H.5 and H.6, we have the bi-doublet of the form:

$$\Phi_1 = \begin{pmatrix} \phi_1^o & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \qquad \text{(H.7)}$$

# Appendix I

# Free parameters of the Model $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

## I.1 Standard Model $SU(2)_L \otimes U(1)_Y$

Remind that many popular science books and articles mention that the Standard Model of particle physics, the model that unifies three of the fundamental forces and describes all matter in the form of quarks and leptons, has about 18 free parameters that are not predicted by the theory.

Very few popular accounts actually tell you what these parameters are. So here they are, in no particular order:

- The so-called fine structure constant, $\alpha$, which (depending on your point of view) defines either the coupling strength of electromagnetism or the magnitude of the electron charge;
- The Weinberg angle or weak mixing angle $\theta_w$ that determines the relationship between the coupling constant of electromagnetism and that of the weak interaction;
- The coupling constant $g_3$ of the strong interaction;
- The electroweak symmetry breaking energy scale (or the Higgs potential vacuum expectation value, V.E.V) $v$;
- The Higgs potential coupling constant $\lambda$ or alternatively, the Higgs mass; However, since 2012 this quantity is known.
- The three mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ and the CP-violating phase $\delta_{13}$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which determines how quarks of various flavor can mix when they interact;
- Nine Yukawa coupling constants that determine the masses of the nine charged fermions (six quarks, three charged leptons).

## I.2 Model with Gauge Symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

In our left-right model we can set 87 free parameters. These are shown in the following items:

- The so-called constants, $g_L$, $g_R$ and $g'$, which (depending on your point of view) defines either the coupling strength of weak and electromagnetism.

- The weak mixing angle $\theta$ that determines the relationship between the coupling constant of electromagnetism and that of the weak interaction;

- The coupling constant $g_3$ of the strong interaction;

- Within the electroweak symmetry breaking energy scale, we have six VEVs: $k_1$, $k_1'$, $k_2$, $k_2'$, $v_L$ and $v_R$.

- The Higgs potential coupling constants, expression 2.17, we have 58 free parameters, that is: $\mu_{ij}$, $\tilde{\mu}_{ij}$, $\mu_{LR}$, $\lambda_{ij}$, $\tilde{\lambda}_{ij}$, $\lambda'_{ij}$, $\tilde{\lambda}'_{ij}$, $\rho_{ij}$, $\tilde{\rho}_{ij}$, $\Lambda_{ij}$, $\tilde{\Lambda}_{ij}$, $\bar{\Lambda}_{ij}$, $\bar{\Lambda}'_{ij}$, $\Omega_{ij}$, $\Omega'_{ij}$, $\lambda_{LR}$.

- The three mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ and the CP-violating phase $\delta_{13}$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which determines how quarks of various flavor can mix when they interact.

- The three mixing angles $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ and the CP-violating phase $\delta_{13}$ of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which determines how charged leptons and neutrinos of various flavor can mix (neutrino oscillations) when they interact;

- Twelve Yukawa coupling constants that determine the masses of the twelve fermions (six quarks, three charged leptons and three neutrinos.

# Appendix J

# PMNS Matrix using the differents neutrinos masses Hierarchy

## J.1 Inverted Hierarchy

The neutrinos matrix, using the Inverted mass hierarchy ($m_3 \ll m_1 < m_2$) is given by the following matricial expression:

$$\hat{M}_\nu = \begin{pmatrix} 0.0497 & 0 & 0 \\ 0 & 0.0504 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{J.1}$$

The coupling matrix for the neutrinos, $G$, is given by:

$$G = \frac{\sqrt{2}}{k_1} U_{PMNS} \hat{M}_\nu U^\dagger_{PMNS} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}, \tag{J.2}$$

Taking into account the numerical values of the mixing angles and the phase angle written in the Particle Data Group, we get:

$$\begin{aligned} s^2_{12} &= 0.307 \\ s^2_{23} &= 0.512 \\ s^2_{13} &= 0.0218 \\ \delta &= 0 \\ k_1 &= 2\,\mathrm{GeV}, \end{aligned}$$

The coupling matrix elements G are given by the following:

$$\begin{aligned} G_{11} &= 3.4531 \times 10^{-11} \\ G_{12} &= -3.1673 \times 10^{-12} \\ G_{13} &= -4.0272 \times 10^{-12} \\ G_{21} &= -3.1673 \times 10^{-12} \\ G_{22} &= 2.0697 \times 10^{-11} \\ G_{23} &= -1.7175 \times 10^{-11} \\ G_{31} &= -4.0272 \times 10^{-12} \\ G_{32} &= -1.7175 \times 10^{-11} \\ G_{33} &= 1.5553 \times 10^{-11} \end{aligned}$$

## J.2 Cuasi-degenerated Hierarchy

The neutrinos matrix, using the Cuasi-degenerated hierarchy ($m_1 \cong m_2 \cong m_3 \cong m_0$, $m_0 \gtrsim 0.10\, eV$) is given by the following matricial expression:

$$\hat{M}_\nu = \begin{pmatrix} 0.10 & 0 & 0 \\ 0 & 0.10 & 0 \\ 0 & 0 & 0.10 \end{pmatrix} \tag{J.3}$$

The coupling matrix for the neutrinos, $G$, is given by:

$$G = \frac{\sqrt{2}}{k_1} U_{PMNS} \hat{M}_\nu U^\dagger_{PMNS} = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}, \tag{J.4}$$

Taking into account the numerical values of the mixing angles and the phase angle written in the Particle Data Group, we get:

$$\begin{aligned} s^2_{12} &= 0.307 \\ s^2_{23} &= 0.512 \\ s^2_{13} &= 0.0218 \\ \delta &= 1.38\,\pi \\ k_1 &= 2\,\text{GeV}, \end{aligned}$$

The coupling matrix elements G are given by the following:

$$\begin{aligned} G_{11} &= 7.0707 \times 10^{-11} \\ G_{12} &= -2.4533 \times 10^{-27}\, i + 1.8399 \times 10^{-27} \\ G_{13} &= -1.2266 \times 10^{-27}\, i - 2.4533 \times 10^{-27} \\ G_{21} &= 2.4533 \times 10^{-27}\, i + 1.8399 \times 10^{-27} \\ G_{22} &= 7.0711 \times 10^{-11} \\ G_{23} &= -9.8131 \times 10^{-27} + 6.1332 \times 10^{-28}\, i \\ G_{31} &= 2.4533 \times 10^{-27}\, i - 1.8399 \times 10^{-27} \\ G_{32} &= -9.8131 \times 10^{-27} - 6.1332 \times 10^{-28}\, i \\ G_{33} &= 7.0711 \times 10^{-11} \end{aligned}$$

Si $\delta = 0$, we have:

$$\begin{aligned} G_{11} &= 7.0711 \times 10^{-11} \\ G_{12} &= -6.1332 \times 10^{-27} \\ G_{13} &= -4.9065 \times 10^{-27} \\ G_{21} &= -1.2266 \times 10^{-27} \\ G_{22} &= 7.0711 \times 10^{-11} \\ G_{23} &= -9.8131 \times 10^{-27} \\ G_{31} &= -4.9065 \times 10^{-27} \\ G_{32} &= 0.0 \\ G_{33} &= 7.0711 \times 10^{-11} \end{aligned}$$

## J.3 Normal Hierarchy

The neutrinos matrix, using the normal mass hierarchy ($m_1 \ll m_2 < m_3$) is given by the following matricial expression:

$$\hat{M}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.0086 & 0 \\ 0 & 0 & 0.0504 \end{pmatrix} \tag{J.5}$$

$$\begin{aligned}
G_{11} &= \frac{\sqrt{2}}{k_1}\left(|\Delta m^2_{31}|^{1/2}\, s^2_{13} + (\Delta m^2_{21})^{1/2} s^2_{12}\, c^2_{13}\right), \\
G_{12} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta}) + |\Delta m^2_{31}|^{1/2}\, c_{13}s_{13}s_{23}\, e^{-i\delta}\right), \\
G_{13} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, s_{12}c_{13}(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta}) + |\Delta m^2_{31}|^{1/2}\, c_{13}s_{13}c_{23}\, e^{-i\delta}\right), \\
G_{21} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c_{13}s_{13}s_{23}\, e^{i\delta}\right), \\
G_{22} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, (c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta})(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c^2_{13}s^2_{23}\right), \quad \text{(J.6)} \\
G_{23} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, (-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta})(c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c^2_{13}c_{23}s_{23}\right), \\
G_{31} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, s_{12}c_{13}(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c_{13}s_{13}c_{23}\, e^{i\delta}\right), \\
G_{32} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, (c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{-i\delta})(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c^2_{13}c_{23}s_{23}\right), \\
G_{33} &= \frac{\sqrt{2}}{k_1}\left((\Delta m^2_{21})^{1/2}\, (-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{-i\delta})(-c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\delta}) + |\Delta m^2_{31}|^{1/2}\, c^2_{13}c^2_{23}\right),
\end{aligned}$$

Taking into account the numerical values of the mixing angles and the phase angle written in the Particle Data Group (2020):

$$\begin{aligned}
s^2_{12} &= 0.307, \\
s^2_{23} &= 0.512, \\
s^2_{13} &= 0.0218, \\
\delta &= 1.37\,\pi, \\
k_1 &= 2\,\text{GeV},
\end{aligned} \tag{J.7}$$

Replacing the experimental values, hence the coupling matrix elements G, (4.10), are given by the following:

$$\begin{aligned}
G_{11} &= 2.606200\times 10^{-12}, \\
G_{12} &= 3.252116\times 10^{-12}\, i + 5.306357\times 10^{-13}, \\
G_{13} &= 3.174979\times 10^{-12}\, i - 3.358971\times 10^{-12}, \\
G_{21} &= -3.252116\times 10^{-12}\, i + 5.306357\times 10^{-13}, \\
G_{22} &= 2.016160\times 10^{-11}, \\
G_{23} &= 1.540466 - 3.800792\times 10^{-13}\, i, \\
G_{31} &= -3.174979\times 10^{-12}\, i - 3.358971\times 10^{-12}, \\
G_{32} &= 1.540466\times 10^{-11} + 3.800792\times 10^{-13}\, i, \\
G_{33} &= 1.909292\times 10^{-11},
\end{aligned} \tag{J.8}$$

### J.3.1 Normal Hierarchy in the case of $g_L \neq g_R$

I propose the following unitary matrix like a PMNS matrix:

$$U_L = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{-i\,\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}\, e^{i\,\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\,\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}\, e^{i\,\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\,\delta} & c_{13}c_{23} \end{pmatrix} \tag{J.9}$$

$$U_R = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}\, e^{-i\,\delta_R} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}\, e^{i\,\delta_R} & C_{12}C_{23} - S_{12}S_{13}S_{23}\, e^{i\,\delta_R} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}\, e^{i\,\delta_R} & -C_{12}S_{23} - S_{12}S_{13}C_{23}\, e^{i\,\delta_R} & C_{13}C_{23} \end{pmatrix} \tag{J.10}$$

The coupling matrix for the neutrinos, $G$, is given by:

$$G = \frac{\sqrt{2}}{k_1} U_L \hat{M}_\nu U_R^\dagger = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}, \tag{J.11}$$

Considering $U_R = U_{CKM}$, and using the values of the CKM matrix elements according to the PDG, we have:

$$\begin{aligned}
U_R &= \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}\, e^{-i\,\delta_R} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}\, e^{i\,\delta_R} & C_{12}C_{23} - S_{12}S_{13}S_{23}\, e^{i\,\delta_R} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}\, e^{i\,\delta_R} & -C_{12}S_{23} - S_{12}S_{13}C_{23}\, e^{i\,\delta_R} & C_{13}C_{23} \end{pmatrix} = \\
&= \begin{pmatrix} 0.97446 & 0.22452 & 0.00365 \\ 0.22438 & 0.97359 & 0.04214 \\ 0.00896 & 0.04133 & 0.999105 \end{pmatrix}
\end{aligned} \tag{J.12}$$

Taking into account the numerical values of the mixing angles and the phase angle written in the Particle Data Group, we get:

$$\begin{aligned}
s_{12}^2 &= 0.307 \\
s_{23}^2 &= 0.512 \\
s_{13}^2 &= 0.0218 \\
\delta &= 0\,\pi \\
k_1 &= 2\,\mathrm{GeV},
\end{aligned}$$

we have:

$$\begin{aligned}
G_{11} &= 7.55106 \times 10^{-13} \\
G_{12} &= 3.41187 \times 10^{-12} \\
G_{13} &= 5.35639 \times 10^{-12} \\
G_{21} &= 8.80819 \times 10^{-13} \\
G_{22} &= 4.42423 \times 10^{-12} \\
G_{23} &= 2.31083 \times 10^{-11} \\
G_{31} &= -7.31455 \times 10^{-13} \\
G_{32} &= -2.46844 \times 10^{-12} \\
G_{33} &= 2.65554 \times 10^{-11}
\end{aligned}$$

### J.3.2 Normal Hierarchy Bi-unitary transformation Matrix

According to the left-right gauge groups, the idea of regarding two differents unitary matrices, each of one relating to the left and right symmetry, we may have the following:

$$G = \frac{\sqrt{2}}{k_1} U_L \hat{M}_\nu U_R^\dagger = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix}, \tag{J.13}$$

where:

$$U_L = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{-i\,\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}\, e^{i\,\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}\, e^{i\,\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}\, e^{i\,\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}\, e^{i\,\delta} & c_{13}c_{23} \end{pmatrix} \tag{J.14}$$

$$\hat{M}_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.0086 & 0 \\ 0 & 0 & 0.0506 \end{pmatrix} \tag{J.15}$$

$$U_R = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}\, e^{-i\,\delta_R} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}\, e^{i\,\delta_R} & C_{12}C_{23} - S_{12}S_{13}S_{23}\, e^{i\,\delta_R} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}\, e^{i\,\delta_R} & -C_{12}S_{23} - S_{12}S_{13}C_{23}\, e^{i\,\delta_R} & C_{13}C_{23} \end{pmatrix} \tag{J.16}$$

$$\begin{aligned}
s_{12}^2 &= 0.307, \\
s_{23}^2 &= 0.512, \\
s_{13}^2 &= 0.0218, \\
\delta &= 1.37\,\pi \\
k_1 &= 2\,\mathrm{GeV},
\end{aligned}$$

$$
\begin{aligned}
G_{11} &= 4.908399\times10^{-13}\,i\,C_{12}S_{13}S_{23}e^{i\delta_R}+2.141357\times10^{-12}\,C_{12}S_{13}S_{23}e^{i\delta_R}+0.00328\,S_{12}C_{13}\\
G_{12} &= -0.00327S_{12}S_{13}S_{23}\,e^{-i\delta_R}+i\,0.00485\,C_{13}S_{23}-0.00192\,C_{13}S_{23}+0.00328\,C_{12}C_{23}\\
G_{13} &= -0.003278\,S_{12}S_{13}C_{23}\,e^{-i\delta_R}-0.00327C_{12}S_{23}+i\,0.00485C_{13}C_{23}-0.00192C_{13}C_{23}\\
G_{21} &= 0.02298\,S_{13}\,e^{i\delta_R}+2.94546\times10^{-4}i\,S_{12}C_{13}+0.00398\,S_{12}C_{13}\\
G_{22} &= -2.94546\times10^{-4}\,i\,S_{12}S_{13}S_{23}e^{-i\delta_R}-0.00398\,S_{12}S_{13}S_{23}e^{-i\delta_R}+0.0229C_{13}S_{23}+\\
&+ 2.9455\times10^{-4}i\,C_{12}C_{23}+0.00398C_{12}C_{23}\\
G_{23} &= -2.94546\times10^{-4}\,i\,S_{12}S_{13}C_{23}\,e^{-i\delta_R}-0.003983\,S_{12}S_{13}C_{23}\,e^{-i\delta_R}-2.94546\times10^{-4}\,iC_{12}S_{23}+\\
&- 0.003983\,C_{12}S_{23}+0.02298\,C_{13}C_{23}\\
G_{31} &= 0.02673\,S_{13}\,e^{i\delta_R}+3.42604\times10^{-4}i\,S_{12}C_{13}-0.00318\,S_{12}C_{13}\\
G_{32} &= -3.42605\times10^{-4}\,i\,S_{12}S_{13}S_{23}e^{-i\delta_R}+0.003188\,S_{12}S_{13}S_{23}e^{-i\delta_R}+0.02673C_{13}S_{23}+\\
&+ 3.42604\times10^{-4}i\,C_{12}C_{23}-0.00318C_{12}C_{23}\\
G_{33} &= -3.42604\times10^{-4}\,i\,S_{12}S_{13}C_{23}\,e^{-i\delta_R}+0.003188\,S_{12}S_{13}C_{23}\,e^{-i\delta_R}-3.40604\times10^{-4}\,iC_{12}S_{23}+\\
&+ 0.003188\,C_{12}S_{23}+0.02673\,C_{13}C_{23}
\end{aligned}
$$

# Appendix K

# Explicit parity violation models with left-right gauge symmetries

## K.1 Introduction

This appendix is based on another article published in the Journal of Physics G (J. Phys. G: Nucl. Part. Phys. 48 (2021) 085010 (16pp)). The basic asumptions of the left-right symmetric models is the electroweak gauge $G^{P}_{LR} \equiv SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes \mathcal{P}$ symmetries is that at under generalized parity $\mathcal{P}$, or charge conjugated $\mathcal{C}$, is that the electroweak interactions are invariant under these symmetries at an energy scale larger than the standard electroweak scale i.e., $\approx 100$ GeV Ref.[4, 5, 28, 7]. As a consequence right-handed neutrinos must exist in nature and neutrino are masssive particles. The smallness of the neutrino masses is related with the breaking of parity if scalar triplets are added being the active neutrinos Majorana particles and the right-handed neutrinos are heavy or, if neutrinos are Dirac fermions only scalar doublets are added and the right- handed neutrinos are related with the Dirac field $\nu = \nu_L + \nu_R$ and the mass term is $m_D \bar{\nu}\nu$. However, this is the case the smallness of the neutrino masses need a fine tuning. The rationale for the parity, or charge conjugation, violation in low energy weak interactions is a well known motivation for the left-right symmetric models or even if the parity violation is trigger before the $SU(2)_R$ symmetry breakdown [6, 27].

Here we will study a model with $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes \mathcal{P}$ gauge symmetry without introducting the parity symmetry at all, i.e., the parity is breakdown explicitly and the left-handed weak interactions have no relation with the right-handed ones. Of course, the $SU(2)_R$ symmetry involves heavy vector bosons in order to be compatible with low energy phenomenology.

## K.2 The Model

Our results do not depend in the representation content of the left-right symmetric or almost symmetric model so we will consider the model in Ref. [7]. with the electroweak gauge symmetry:

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \tag{K.1}$$

without introducing the generalized parity $\mathcal{P}$. Hence, $g_L \neq g_R$ from the very beginning and the model has thre gauge couplings $g_L$, $g_R$ and $g_{B-L} \equiv g'$ of the respective factors. The electric charge operator defined [24, 25]

$$\frac{Q}{|e|} = T_{3L} + T_{3R} + \frac{B-L}{2} \tag{K.2}$$

We omit the $SU(3)_C$ factor because is as in the Standard Model. We will also consider only the lepton sector transforming as follows:

$$L'_\ell = \left( \begin{array}{c} \nu^\ell_L \\ \ell' \end{array} \right)_L \sim (\mathbf{2}_L, \mathbf{1}_R. -\tfrac{1}{2}), \quad R'_\ell = \left( \begin{array}{c} \nu^\ell_L \\ \ell' \end{array} \right)_R \sim (\mathbf{1}_L, \mathbf{2}_R. -\tfrac{1}{2}) \tag{K.3}$$

With $\ell = e, \mu, \tau$ and the primed states in (K.3) are symmetry eigenstates.

The scalar sector consists one or more bi-doublets transforming as $(\mathbf{2}_L, \mathbf{2}_R, 0)$:

$$\Phi = \left( \begin{array}{cc} \phi^0_1 & \phi^+_2 \\ \phi^-_1 & \phi^0_2 \end{array} \right), \tag{K.4}$$

to generate the fermion masses and only one doublet $\chi_R \sim (\mathbf{1}, \mathbf{2}, +1)$ or one triplet $\Delta_R \sim (\mathbf{1}, \mathbf{3}, +2)$ to break the gauge symmetry down to $U(1)_Q$. Here we will consider, for simplicity, the cases of one bi-doublet and one doublet.

$$\chi_R = \left( \begin{array}{c} \chi^+_R \\ \chi^0_R \end{array} \right) \tag{K.5}$$

to break completely the gauge symmetry to $U(1)_Q$ A doblet $\chi_L \sim (\mathbf{2}, \mathbf{1}, +1)$ or a triplet $\Delta_L \sim (\mathbf{3}, \mathbf{1}, +2)$ can be added but are not mandatory in this case. They can be added to have a inert doublet or triplet. A triplet $\Delta_R \sim (\mathbf{1}, \mathbf{3}, +2)$ may be added to give a Majorana mass term to right-handed neutrino and implement the type-I seesaw mechanism. Here, we will consider the minimal case of a bi-doublet in (K.4) and the doublet in (K.5) since they are enough to shown the main feauture of the model.

The vacuum expectation values, (VEVs), are:

$$\langle \Phi_1 \rangle = \tfrac{1}{\sqrt{2}} \left( \begin{array}{cc} k_1 & 0 \\ 0 & k'_1 \end{array} \right), \quad \langle \chi_R \rangle = \tfrac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_R \end{array} \right), \tag{K.6}$$

# K.3 Gauge Bosons Mass Eigenstates

The covariant derivative for the bi-doublets $\Phi_i$ with $i = 1, 2$ and for the doublet $\chi_R$ are given by:

$$\mathcal{D}_\mu \Phi = \partial_\mu \Phi + i \left[ g_L \frac{\vec{\tau}}{2} \cdot \vec{W}_L \Phi - g_R \Phi \frac{\vec{\tau}}{2} \cdot \vec{W}_R \right], \tag{K.7}$$

$$\mathcal{D}_\mu \chi_R = \left( \partial_\mu + i g_R \frac{\vec{\tau}}{2} \cdot \vec{W}_R - i g' B_\mu \right) \chi_R, \tag{K.8}$$

where we have considered $g_L \neq_R$. The VEVs are given in the equations (K.6).

## K.3.1 Charged Bosons mass matrix

We obtain

$$\mathcal{M}_{CB} = \left( \begin{array}{cc} \frac{g_L^2}{4}(k_1^2 + k_1'^2 + k_2^2 + k_2'^2 + v_L^2) & -\frac{g_L g_R}{2}(k_1 k_1' + k_2 k_2') \\ -\frac{g_L g_R}{2}(k_1 k_1' + k_2 k_2') & \frac{g_L^2}{4}(k_1^2 + k_1'^2 + k_2^2 + k_2'^2 + v_R^2) \end{array} \right)$$

Regarding $v_L = k_2 = k_2' = 0$, we have:

$$\mathcal{M}_{CB} = \left( \begin{array}{cc} \frac{g_L^2}{4} K^2 & -\frac{g_L g_R}{2} \bar{K}^2 \\ -\frac{g_L g_R}{2} \bar{K}^2 & \frac{g_L^2}{4}(K^2 + v_R^2) \end{array} \right)$$

where: $K^2 = k_1^2 + k_1'^2$, $\bar{K}^2 = k_1 k_1'$, and the charged gauge bosons masses are given by:

$$M_{W_1}^2 = \frac{1}{8}\left(K^2(g_L^2 + g_R^2) + g_R^2 v_R^2 - \sqrt{\Delta'}\right), \tag{K.9}$$

$$M_{W_2}^2 = \frac{1}{8}\left(K^2(g_L^2 + g_R^2) + g_R^2 v_R^2 + \sqrt{\Delta'}\right), \tag{K.10}$$

where: $\Delta' = 16\, g_L^2 g_R^2 \bar{K}^4 + \left[g_R^2 v_R^2 + K^2(g_R^2 - g_L^2)\right]^2$.

The masses eigenstates as a function of the symmetry eigenstates are given by:

$$W_{1\mu}^+ = c_\xi\, W_L^+ + s_\xi\, W_R^+, \tag{K.11}$$

$$W_{2\mu}^+ = -s_\xi\, W_L^+ + c_\xi\, W_R^+, \tag{K.12}$$

where:

$$s_\xi = \frac{4\, g_L g_R\, \bar{K}^2}{\sqrt{16 g_L^2 g_R^2 \bar{K}^4 + Y^2}}, \tag{K.13}$$

$$c_\xi = \frac{Y}{\sqrt{16 g_L^2 g_R^2 \bar{K}^4 + Y^2}}, \tag{K.14}$$

$$Y = K^2(g_R^2 - g_L^2) + g_R^2 v_R^2 + \sqrt{\Delta'}, \tag{K.15}$$

Making $v_R^2 \gg X^2$, where $X$ is another VEV, we have:

$$M_{W_1}^2 \approx \frac{1}{4} g_L^2 K^2, \tag{K.16}$$

$$M_{W_2}^2 \approx \frac{1}{4} g_R^2 v_R^2, \tag{K.17}$$

We can observe that $M_{W_2}^2 \gg M_{W_1}^2$

### K.3.2 Neutral Bosons mass matrix

$$\mathcal{M}_{NB} = \begin{pmatrix} \frac{g_L^2}{4}(K^2 + v_L^2) & -\frac{g_L g_R}{4} K^2 & -\frac{g' g_L}{4} v_L^2 \\ -\frac{g_L g_R}{4} K^2 & \frac{g_R^2}{4}(K^2 + v_R^2) & -\frac{g' g_R}{4} v_R^2 \\ -\frac{g' g_L}{4} v_L^2 & -\frac{g' g_R}{4} v_R^2 & \frac{g'^2}{4}(v_L^2 + v_R^2) \end{pmatrix},$$

remember that $K^2 \equiv k_1^2 + k_1'^2 + k_2^2 + k_2'^2$.

Making $v_L = k_2 = k_2' = 0$, we have:

$$\mathcal{M}_{NB} = \begin{pmatrix} \frac{g_L^2}{4} K^2 & -\frac{g_L g_R}{4} K^2 & 0 \\ -\frac{g_L g_R}{4} K^2 & \frac{g_R^2}{4}(K^2 + v_R^2) & -\frac{g' g_R}{4} v_R^2 \\ 0 & -\frac{g' g_R}{4} v_R^2 & \frac{g'^2}{4} v_R^2 \end{pmatrix},$$

Diagonalizing we have:

$$M_{Z_1}^2 = \frac{1}{8}\left[K^2(g_L^2 + g_R^2) + v_R^2(g'^2 + g_R^2) - \sqrt{\Delta}\right], \tag{K.18}$$

$$M_{Z_2}^2 = \frac{1}{8}\left[K^2(g_L^2 + g_R^2) + v_R^2(g'^2 + g_R^2) + \sqrt{\Delta}\right], \tag{K.19}$$

$$M_{A_\mu}^2 = 0. \tag{K.20}$$

Where:

$$\Delta = (g'^2 + g_R^2)^2 v_R^4 + (g_R^2 + g_L^2)^2 K^4 + 2K^2 v_R^2 \left[g_R^4 - g_R^2 g_L^2 - g_R^2 g'^2 - g_L^2 g'^2\right]$$

Diagonalizing we have:

$$M_{Z_1}^2 = \frac{1}{8}\left[K^2(g_L^2 + g_R^2) + v_R^2(g'^2 + g_R^2) - \sqrt{\Delta}\right], \tag{K.21}$$

$$M_{Z_2}^2 = \frac{1}{8}\left[K^2(g_L^2 + g_R^2) + v_R^2(g'^2 + g_R^2) + \sqrt{\Delta}\right], \tag{K.22}$$

$$M_{A_\mu}^2 = 0. \tag{K.23}$$

Where:

$$\Delta = (g'^2 + g_R^2)^2 v_R^4 + (g_R^2 + g_L^2)^2 K^4 + 2K^2 v_R^2 \left[g_R^4 - g_R^2 g_L^2 - g_R^2 g'^2 - g_L^2 g'^2\right] \tag{K.24}$$

Making $v_R^2 \gg X^2$, where $X$ is another VEV, in the same way as the previous section, we have:

$$M_{Z_1}^2 \approx \frac{1}{4}\frac{(g_L^2 g_R^2 + [g_R^2 + g_L^2]g'^2)K^2}{g'^2 + g_R^2}, \tag{K.25}$$

$$M_{Z_2}^2 \approx \frac{1}{4}(g_R^2 + g'^2)v_R^2, \tag{K.26}$$

We can observe that $M_{Z_2}^2 \gg M_{Z_1}^2$. Notice that for $v_R \gg X$ we also have:

$$M_{Z_1} \approx \frac{M_{W_1}}{\cos\theta} + \mathcal{O}\left(X^2/v_R^2\right) \tag{K.27}$$

After diagonalizing the neutral matrix by an orthogonal matrix, $\mathcal{O}^T \mathcal{M}_{NB}^2 \mathcal{O} = \hat{M} = \left(0, m_{Z_1}^2, m_{Z_2}^2\right)$, we obtain the following symmetry eigenstates as a function of masses eigenstates:

$$\begin{pmatrix} W_\mu^{3L} \\ W_\mu^{3R} \\ B_\mu \end{pmatrix} = \begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix} \begin{pmatrix} A^\mu \\ Z_1^\mu \\ Z_2^\mu \end{pmatrix}, \tag{K.28}$$

where:

$$n_{11} = 1/N_1, \tag{K.29}$$

$$n_{21} = \frac{g_L}{N_1\, g_R}, \tag{K.30}$$

$$n_{31} = \frac{g_L}{N_1\, g'}, \tag{K.31}$$

$$n_{12} = 1/N_2, \tag{K.32}$$

$$n_{22} = \frac{a_1}{N_2}, \tag{K.33}$$

$$n_{32} = \frac{b_1}{N_2}, \tag{K.34}$$

$$n_{13} = 1/N_3, \tag{K.35}$$

$$n_{23} = \frac{a_2}{N_3}, \tag{K.36}$$

$$n_{33} = \frac{b_2}{N_3}, \tag{K.37}$$

Moreover:

$$
\begin{aligned}
a_1 &= \frac{-(g_R^2 - g_L^2)K^2 - (g'^2 + g_R^2)v_R^2 + \sqrt{\Delta}}{2g_L g_R K^2}, && \text{(K.38)}\\
b_1 &= g'\frac{-(g_R^2 + g_L^2)K^2 + (g'^2 + g_R^2)v_R^2 - \sqrt{\Delta}}{2g_L g_R^2 K^2}, && \text{(K.39)}\\
a_2 &= \frac{-(g_R^2 - g_L^2)K^2 - (g'^2 + g_R^2)v_R^2 - \sqrt{\Delta}}{2g_L g_R K^2}, && \text{(K.40)}\\
b_2 &= g'\frac{-(g_R^2 + g_L^2)K^2 + (g'^2 + g_R^2)v_R^2 + \sqrt{\Delta}}{2g_L g_R^2 K^2}, && \text{(K.41)}\\
N_1 &= \sqrt{1 + (\frac{g_L}{g_R})^2 + (\frac{g_L}{g'})^2}, && \text{(K.42)}\\
N_2 &= \sqrt{1 + a_1^2 + b_1^2}, && \text{(K.43)}\\
N_3 &= \sqrt{1 + a_2^2 + b_2^2}, && \text{(K.44)}
\end{aligned}
$$

and $\Delta$ is given by the expression (K.24). The eigenstates are normalized. These are exact results.

## K.4 Lepton-Vector Boson Interactions

When $g_L \neq g_R$ the covariant derivatives in the lepton sector are given by:

$$
\begin{aligned}
\mathcal{D}_\mu L &= \left(\partial_\mu + i\frac{g_L}{2}\vec{\tau}\cdot\vec{W}_{L\mu} - i\frac{g'}{2}B_\mu\right)L, && \text{(K.45)}\\
\mathcal{D}_\mu R &= \left(\partial_\mu + i\frac{g_R}{2}\vec{\tau}\cdot\vec{W}_{R\mu} - i\frac{g'}{2}B_\mu\right)R, && \text{(K.46)}
\end{aligned}
$$

and similarly for quarks.

From projection on the photon field in Eq. (K.28) y (K.45) we may obtain the electric charge:

$$\mathcal{Q}_{\ell_L} = \mathcal{Q}_{\ell_R} \equiv -e = \frac{g_L g_R g'}{\Omega} \qquad \text{(K.47)}$$

or

$$e^2 = \frac{g_L^2 g_R^2 g'^2}{(g_L^2 + g_R^2)g'^2 + g_L^2 g_R^2} = \frac{g_L^2 g_R^2 g'^2}{\Omega^2}, \qquad \text{(K.48)}$$

where:

$$\Omega = \sqrt{(g_L^2 + g_R^2)\,g'^2 + g_L^2 g_R^2}.$$

Similarly for quarks.

From (K.48) we have:

$$\frac{1}{e^2} = \frac{1}{g_L^2} + \frac{1}{g_R^2} + \frac{1}{g'^2} \qquad \text{(K.49)}$$

Notice that in the present case only $g_L$ can be related to the $g$ coupling constant of the $SU(2)_L$ electroweak standard model, while $g_R$ and $g'$ have no relation with $g$ and $g_Y$ and should be phenomenological constrained.

Using Eq. (K.28) y (K.47) we can write it like this:

$$\begin{pmatrix} W_\mu^{3L} \\ W_\mu^{3R} \\ B_\mu \end{pmatrix} = \begin{pmatrix} \frac{g_R g'}{\Omega} & \frac{1}{N_2} & \frac{1}{N_3} \\ \frac{g_L g'}{\Omega} & \frac{a_1}{N_2} & \frac{a_2}{N_3} \\ \frac{g_R g_L}{\Omega} & \frac{b_1}{N_2} & \frac{b_2}{N_3} \end{pmatrix} \begin{pmatrix} A^\mu \\ Z_1^\mu \\ Z_2^\mu \end{pmatrix}, \tag{K.50}$$

Note that:

$$\mathcal{Q}_{\nu_L} = \mathcal{Q}_{\nu_R} = 0 \tag{K.51}$$

Using the appropriate projection of $W_{3L}$, $W_{3R}$ and $B_\mu$ upon the photon field it is not necessary to impose extra constraints over the matrix elements of the matrix in Eq. (K.28) or (K.50). In Ref. [12], the author began with a general orthogonal matrix relating $W_{3L}$, $W_{3R}$ and $B_\mu$ with $A$, $Z_1$, $Z_2$ and for this reason extra constraints had to be imposed on the matrix elements. However, now it is possible to define three angles $\theta_{ij}$ of an orthogonal $3 \times 3$ matrix:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - s_{12}s_{13}c_{23} & c_{13}c_{23} \end{pmatrix}, \tag{K.52}$$

such that:

$$\begin{pmatrix} \frac{g_R g'}{\sqrt{\Omega}} & \frac{1}{N_2} & \frac{1}{N_3} \\ \frac{g_L g'}{\sqrt{\Omega}} & \frac{a_1}{N_2} & \frac{a_2}{N_3} \\ \frac{g_R g_L}{\sqrt{\Omega}} & \frac{b_1}{N_2} & \frac{b_2}{N_3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - s_{12}s_{13}c_{23} & c_{13}c_{23} \end{pmatrix} \tag{K.53}$$

Comparing both matrices, we obtain:

$$\begin{aligned} c_{12}c_{13} &= \frac{g_R g'}{\sqrt{\Omega}}, \\ s_{12}c_{13} &= \frac{1}{N_2}, \\ s_{13} &= \frac{1}{N_3}, \\ -s_{12}c_{23} - c_{12}s_{13}s_{23} &= \frac{g_L g'}{\sqrt{\Omega}}, \\ c_{12}c_{23} - s_{12}s_{13}s_{23} &= \frac{a_1}{N_2}, \\ c_{13}s_{23} &= \frac{a_2}{N_3}, \\ s_{12}s_{23} - c_{12}s_{13}c_{23} &= \frac{g_R g_L}{\sqrt{\Omega}}, \\ -c_{12}s_{23} - s_{12}s_{13}c_{23} &= \frac{b_1}{N_2}, \\ c_{13}c_{23} &= \frac{b_2}{N_3}, \end{aligned} \tag{K.54}$$

Notice that if we assume from the very start that the matrix $\mathcal{O}$ in (K.50) is an arbitrary orthogonal matrix, we obtain, for instance, for neutrinos:

$$\begin{aligned} \mathcal{Q}_{\nu_L} &= g_L c_{12}c_{13} - g'(s_{12}s_{23} - c_{12}s_{13}c_{23}) \\ \mathcal{Q}_{\nu_R} &= -g_L(s_{12}c_{23} + c_{12}s_{13}s_{23}) - g'(s_{12}s_{23} - c_{12}s_{13}c_{23}) \neq 0, \end{aligned} \tag{K.55}$$

Hence $\mathcal{Q}_{\nu_L} \neq \mathcal{Q}_{\nu_R} \neq 0$, for arbitrary angles. Similarly all left-handed fields have electric charge which is different from the charge of the right-handed components. Only when the massles vector field (photon) has been identified, it is possible to put the constraints in Eq. (K.54) like has been done in [61, 62].

In left-right symmtric model, the invariance under a generalized parity symmetry implies: $g_L = g_R$ and we have from Eq. (K.49):

$$\frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2}, \tag{K.56}$$

and the charged leptons electric charge is given by:

$$e_q = q\frac{g\,g'}{\sqrt{g^2 + 2g'^2}}, \tag{K.57}$$

with $q = -1, 2/3, -1/3$ for charged leptons, u-like and d-like quarks, respectively.

From (K.57):

$$e = g\sin\theta, \qquad e = g'\sqrt{\cos 2\theta}, \tag{K.58}$$

where:

$$\sin\theta = \frac{g'}{\sqrt{g^2 + g'^2}}, \qquad \cos 2\theta = \frac{g}{\sqrt{g^2 + 2g'^2}} \tag{K.59}$$

Arises in left-right symmetric model and again we obtain the relation in Eq. (K.56). However, it is not possible to mantain the equality of $g_L$ and $g_R$ for all the energy range, but only at a give energy, say at the $\mathcal{Z}$-pole because the running of both coupling constants are different since they feel different degrees of freedom [6, 27].

### K.4.1 Charged Interactions

We know from the lagrangian:

$$\mathcal{L}_W = -\frac{1}{2}\left[g_L\bar{\nu}_L\gamma^\mu V^L_{PMNS}\ell_L W^+_{L\mu} + g_R\bar{\nu}_R\gamma^\mu V^R_{PMNS}\ell_R W^+_{R\mu}\right] + H.C., \tag{K.60}$$

Besides the fact that $g_L \neq g_R$, in the general case where the Yukawa couplings are complex the right-handed CKM matrix is different from the left-handed one. This case was considered in Ref.[16]. However, if we neglect CP violation and consider G and F real as well the VEV $k_1$, we have that $V^L_{PMNS} = V^R_{PMNS} \equiv V_{PMNS}$. Similarly in the quark sector.

### K.4.2 Neutral Interactions

Neutral interactions between the fermions and the neutral vector bosons are the following: The electromagnetic interactions had already been confirmed, the charged fermions has the correct interaction with the photons. See Eq. (K.47). The interactions with the massive neutral vector bosons are given by: Next, we parametrize the neutral interactions of a fermion $i$ with the $Z_{1\mu}$ and $Z_{2\mu}$ neutral bosons as follows:

$$\mathcal{L}_{NC} = -\frac{g}{2\cos\theta}\sum_i \bar{\psi}_i\gamma^\mu\left[\left(g^i_V - g^i_A\gamma^5\right)Z_{1\mu} + \left(f^i_V - f^i_A\gamma^5\right)Z_{2\mu}\right]\psi_i \tag{K.61}$$

We will define:

$$g_V^f = \frac{1}{2}(a_L^f + a_R^f), \qquad g_A^f = \frac{1}{2}(a_L^f - a_R^f), \tag{K.62}$$

where $a_L^f$ and $a_R^f$ are the couplings of the left and right-handed comoponents of a fermion $\psi_i$. Similarly couplings $f_{L,R}^i$ and the respective $f_{V,A}^i$ are defined.

Using (K.28), we obtain for leptons:

$$\begin{aligned}
a_L^\nu &= \frac{g_L}{N_2}(1 - \delta_L b_1), \\
a_R^\nu &= \frac{g_R}{N_2}(a_1 - \delta_R b_1), \\
a_L^\ell &= -\frac{g_L}{N_2}(1 + \delta_L b_1), \\
a_R^\ell &= -\frac{g_R}{N_2}(a_1 + \delta_R b_1),
\end{aligned} \tag{K.63}$$

with $\delta_{L(R)} = g'/g_{L(R)}$ or:

$$\begin{aligned}
g_V^\nu &= \frac{g_L}{N_2}(1 + \epsilon a_1 - 2\delta_L b_1), \\
g_V^\ell &= -\frac{g_L}{N_2}(1 + \epsilon a_1 + 2\delta_L b_1), \\
g_A^\nu &= \frac{g_L}{N_2}(1 - \epsilon a_1), \\
g_A^\ell &= -\frac{g_L}{N_2}(1 - \epsilon a_1),
\end{aligned} \tag{K.64}$$

where $\epsilon = g_R/g_L$, and for quarks:

$$\begin{aligned}
a_L^u &= \frac{g_L}{N_2}\left(1 + \frac{1}{6}\delta_L b_1\right), \\
a_R^u &= \frac{g_R}{N_2}\left(a_1 + \frac{1}{6}\delta_L b_1\right), \\
a_L^d &= \frac{g_L}{N_2}\left(-1 + \frac{1}{6}\delta_L b_1\right), \\
a_R^d &= \frac{g_R}{N_2}\left(-a_1 + \frac{1}{6}\delta_L b_1\right),
\end{aligned} \tag{K.65}$$

with:

$$\begin{aligned}
g_V^u &= \frac{g_L}{2N_2}\left[1 + \epsilon a_1 + \frac{1}{6}(\delta_L + \delta_R)b_1\right], \\
g_A^u &= \frac{g_R}{N_2}\left[1 - \epsilon a_1 + \frac{1}{6}(\delta_L - \delta_R)b_1\right], \\
g_V^d &= -\frac{1}{2N_2}\left[1 + \epsilon a_1 - \frac{1}{6}(\delta_L - \delta_R)b_1\right], \\
g_A^d &= \frac{1}{2N_2}\left[-1 + \epsilon a_1 + \frac{1}{6}(\delta_L - \delta_R)b_1\right],,
\end{aligned} \tag{K.66}$$

Notice that when $v_R \to \infty$, we obtain:

$$g_V^\nu, g_A^\nu \to \frac{1}{2}. \tag{K.67}$$

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