It is then possible to compute the change in energy required to take the system from one state to another by:

$$W_m(a) - W_m(b) = \int_b^a i d\lambda - f^e dx$$

where the two states of the system are described by $a = (\lambda_a, x_a)$ and $b = (\lambda_b, x_b)$

If the energy stored in the system is described by two state variables, λ and x, the *total differential* of stored energy is:

$$dW_m = \frac{\partial W_m}{\partial \lambda} d\lambda + \frac{\partial W_m}{\partial x} dx$$

and it is also:

$$dW_m = i d\lambda - f^e dx$$

So that we can make a direct equivalence between the derivatives and:

$$f^e = -\frac{\partial W_m}{\partial x}$$

This generalizes in the case of multiple electrical terminals and/or multiple mechanical terminals. For example, a situation with multiple electrical terminals will have:

$$dW_m = \sum_k i_k d\lambda_k - f^e dx$$

And the case of rotary, as opposed to linear, motion has in place of force f^e and displacement x, torque T^e and angular displacement θ.

In many cases we might consider a system which is electricaly *linear*, in which case inductance is a function only of the mechanical position x.

$$\lambda(x) = L(x)i$$

In this case, assuming that the energy integral is carried out from $\lambda = 0$ (so that the part of the integral carried out over x is zero),

$$W_m = \int_0^\lambda \frac{1}{L(x)} \lambda d\lambda = \frac{1}{2} \frac{\lambda^2}{L(x)}$$

This makes

$$f^e = -\frac{1}{2}\lambda^2 \frac{\partial}{\partial x} \frac{1}{L(x)}$$

Note that this is numerically equivalent to

$$f^e = -\frac{1}{2} i^2 \frac{\partial}{\partial x} L(x)$$

This is true *only* in the case of a linear system. Note that substituting $L(x)i = \lambda$ too early in the derivation produces erroneous results: in the case of a linear system it is a sign error, but in the case of a nonlinear system it is just wrong.

2.1.1 Coenergy

We often will describe systems in terms of inductance rather than its reciprocal, so that current, rather than flux, appears to be the relevant variable. It is convenient to derive a new energy variable, which we will call *co-energy*, by:

$$W'_m = \sum_i \lambda_i i_i - W_m$$

and in this case it is quite easy to show that the energy differential is (for a single mechanical variable) simply:

$$dW'_m = \sum_k \lambda_k di_k + f^e dx$$

so that force produced is:

$$f_e = \frac{\partial W'_m}{\partial x}$$

Consider a simple electric machine example in which there is a single winding on a rotor (call it the *field* winding and a polyphase armature. Suppose the rotor is round so that we can describe the flux linkages as:

$$\begin{aligned}
\lambda_a &= L_a i_a + L_{ab} i_b + L_{ab} i_c + M\cos(p\theta) i_f \\
\lambda_b &= L_{ab} i_a + L_a i_b + L_{ab} i_c + M\cos(p\theta - \frac{2\pi}{3}) i_f \\
\lambda_c &= L_{ab} i_a + L_{ab} i_b + L_a i_c + M\cos(p\theta + \frac{2\pi}{3}) i_f \\
\lambda_f &= M\cos(p\theta) i_a + M\cos(p\theta - \frac{2\pi}{3}) i_b + M\cos(p\theta + \frac{2\pi}{3}) + L_f i_f
\end{aligned}$$

Now, this system can be simply described in terms of coenergy. With multiple excitation it is important to exercise some care in taking the coenergy integral (to ensure that it is taken over a valid path in the multi-dimensional space). In our case there are actually five dimensions, but only four are important since we can position the rotor with all currents at zero so there is no contribution to coenergy from setting rotor position. Suppose the rotor is at some angle θ and that the four currents have values i_{a0}, i_{b0}, i_{c0} and i_{f0}. One of many correct path integrals to take would be:

$$\begin{aligned}
W'_m &= \int_0^{i_{a0}} L_a i_a di_a \\
&+ \int_0^{i_{b0}} \left(L_{ab} i_{a0} + L_a i_b\right) di_b \\
&+ \int_0^{i_{c0}} \left(L_{ab} i_{a0} + L_{ab} i_{b0} + L_a i_c\right) di_c \\
&+ \int_0^{i_{f0}} \left(M\cos(p\theta) i_{a0} + M\cos(p\theta - \frac{2\pi}{3}) i_{b0} + M\cos(p\theta + \frac{2\pi}{3}) i_{c0} + L_f i_f\right) di_f
\end{aligned}$$

The result is:

$$\begin{aligned}
W'_m &= \frac{1}{2} L_a \left(i_{a0}^2 + i_{b0}^2 + i_{co}^2\right) + L_{ab}\left(i_{ao} i_{b0} + i_{ao} i_{c0} + i_{co} i_{b0}\right) \\
&+ M i_{f0} \left(i_{a0}\cos(p\theta) + i_{b0}\cos(p\theta - \frac{2\pi}{3}) + i_{c0}\cos(p\theta + \frac{2\pi}{3})\right) + \frac{1}{2} L_f i_{f0}^2
\end{aligned}$$

Mechanical power is torque times speed:

$$P_m = T\Omega$$

And the sum of the losses is the difference:

$$P_d = P_e - P_m$$

It will sometimes be convenient to employ the fact that, in most machines, dissipation is small enough to approximate mechanical power with electrical power. In fact, there are many situations in which the loss mechanism is known well enough that it can be idealized away. The "thermodynamic" arguments for force density take advantage of this and employ a "conservative" or lossless energy conversion system.

2.1 Energy Approach to Electromagnetic Forces:

Figure 2: Conservative Magnetic Field System

To start, consider some electromechanical system which has two sets of "terminals", electrical and mechanical, as shown in Figure 2. If the system stores energy in magnetic fields, the energy stored depends on the *state* of the system, defined by (in this case) two of the identifiable variables: flux (λ), current (i) and mechanical position (x). In fact, with only a little reflection, you should be able to convince yourself that this state is a single-valued function of two variables and that the energy stored is independent of how the system was brought to this state.

Now, all electromechanical converters have loss mechanisms and so are not themselves conservative. However, the magnetic field system that produces force is, in principle, conservative in the sense that its state and stored energy can be described by only two variables. The "history" of the system is not important.

It is possible to chose the variables in such a way that electrical power *into* this conservative system is:

$$P^e = vi = i\frac{d\lambda}{dt}$$

Similarly, mechanical power *out of* the system is:

$$P^m = f^e\frac{dx}{dt}$$

The difference between these two is the rate of change of energy stored in the system:

$$\frac{dW_m}{dt} = P^e - P^m$$

If the rotor is round so that there is no variation of the stator inductances with rotor position θ, torque is easily given by:

$$T_e = \frac{\partial W'_m}{\partial \theta} = -pMi_{f0}\left(i_{a0}\sin(p\theta) + i_{b0}\sin(p\theta - \frac{2\pi}{3}) + i_{co}\sin(p\theta + \frac{2\pi}{3})\right)$$

2.2 Continuum Energy Flow

At this point, it is instructive to think of electromagnetic energy flow as described by *Poynting's Theorem:*

$$\vec{S} = \vec{E} \times \vec{H}$$

Energy flow $\vec{S}$, called *Poynting's Vector*, describes electromagnetic energy flow in terms of electric and magnetic fields.

To calculate electromagnetic energy flow *into* some volume of space, we can integrate Poyting's Vector over the surface of that volume, and then using the divergence theorem:

$$P = -\oiint \vec{S} \cdot \vec{n} da = -\int_{\text{vol}} \nabla \cdot \vec{S} dv$$

Now, the divergence of the Poynting Vector is, using a vector identity:

$$\begin{aligned} \nabla \cdot \vec{S} &= \nabla \cdot \left(\vec{E} \times \vec{H}\right) = \vec{H} \cdot \nabla \times \vec{E} - \vec{E} \cdot \nabla \times \vec{H} \\ &= -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \vec{J} \end{aligned}$$

The power crossing into a region of space is then:

$$P = \int_{\text{vol}} \left(\vec{E} \cdot \vec{J} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}\right) dv$$

Now, in the absence of material motion, interpretation of the two terms in this equation is fairly simple. The first term describes dissipation:

$$\vec{E} \cdot \vec{J} = |\vec{E}|^2 \sigma = |\vec{J}|^2 \rho$$

The second term is interpreted as rate of change of magnetic stored energy. In the absence of hysteresis it is:

$$\frac{\partial W_m}{\partial t} = \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}$$

Some materials exhibit hysteretic behavior, in which stored energy is not a single valued function of either $\vec{B}$ or $\vec{H}$, and we will consider that case anon.

In the presence of material motion $\vec{v}$, electric field $\vec{E}'$ in a "moving" frame is related to electric field $\vec{E}$ in a "stationary" frame and to magnetic field $\vec{B}$ by:

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

This is a relatavistic expression, so that the qualifiers "moving" and "stationary" are themselves relative. The electric fields are what would be observed in either frame. In MQS systems, the magnetic flux density $\vec{B}$ is the same in both frames.

This last expression is yet another way of describing energy conversion processes in electric machinery, as the component of apparent electric field produced by material motion through a magnetic field, when reacted against by a current, produces energy conversion to mechanical form rather than dissipation.

2.3 Coenergy in Continuous Media

Now, consider a system with not just a multiplicity of circuits but a continuum of current-carrying paths. In that case we could identify the co-energy as:

$$W'_m = \int_{\text{area}} \int \lambda(\vec{a}) d\vec{J} \cdot d\vec{a}$$

where that area is chosen to cut all of the current carrying conductors. This area can be picked to be perpedicular to each of the current filaments since the divergence of current is zero. The flux λ is calculated over a path that coincides with each current filament (such paths exist since current has zero divergence). Then the flux is:

$$\lambda(\vec{a}) = \int \vec{B} \cdot d\vec{n}$$

Now, if we use the vector potential $\vec{A}$ for which the magnetic flux density is:

$$\vec{B} = \nabla \times \vec{A}$$

the flux linked by any one of the current filaments is:

$$\lambda(\vec{a}) = \oint \vec{A} \cdot d\vec{\ell}$$

where $d\vec{\ell}$ is the path around the current filament. This implies directly that the coenergy is:

$$W'_m = \int_{\text{area}} \int_J \oint \vec{A} \cdot d\vec{\ell} d\vec{J} \cdot d\vec{a}$$

Now: it is possible to make $d\vec{\ell}$ coincide with $d\vec{a}$ and be parallel to the current filaments, so that:

$$W'_m = \int_{\text{vol}} \vec{A} \cdot d\vec{J} dv$$

2.3.1 Permanent Magnets

Permanent magnets are becoming an even more important element in electric machine systems. Often systems with permanent magnets are approached in a relatively ad-hoc way, made equivalent to a current that produces the same MMF as the magnet itself.

The constitutive relationship for a permanent magnet relates the magnetic flux density $\vec{B}$ to magnetic field $\vec{H}$ and the property of the magnet itself, the *magnetization* $\vec{M}$.

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right)$$

Now, the effect of the magnetization is to act as if there were a current (called an *amperian current*) with density:

$$\vec{J}^* = \nabla \times \vec{M}$$

Note that this amperian current "acts" just like ordinary current in making magnetic flux density. Magnetic co-energy is:

$$W'_m = \int_{\text{vol}} \vec{A} \; \nabla \times d\vec{M} dv$$

Next, note the vector identity $\nabla \cdot \left(\vec{C} \times \vec{D}\right) = \vec{D} \cdot \left(\nabla \times \vec{C}\right) - \vec{C} \cdot \left(\nabla \times \vec{D}\right)$ Now,

$$W'_m = \int_{\text{vol}} -\nabla \cdot \left(\vec{A} \times d\vec{M}\right) dv + \int_{\text{vol}} \left(\nabla \times \vec{A}\right) \cdot d\vec{M} dv$$

Then, noting that $\vec{B} = \nabla \times \vec{A}$:

$$W'_m = -\oiint \vec{A} \times d\vec{M} d\vec{s} + \int_{\text{vol}} \vec{B} \cdot d\vec{M} dv$$

The first of these integrals (closed surface) vanishes if it is taken over a surface just outside the magnet, where $\vec{M}$ is zero. Thus the magnetic co-energy in a system with only a permanent magnet source is

$$W'_m = \int_{\text{vol}} \vec{B} \cdot d\vec{M} dv$$

Adding current carrying coils to such a system is done in the obvious way.

2.4 Electric Machine Description:

Actually, this description shows a conventional induction motor. This is a very common type of electric machine and will serve as a reference point. Most other electric machines operate in a fashion which is the same as the induction machine or which differ in ways which are easy to reference to the induction machine.

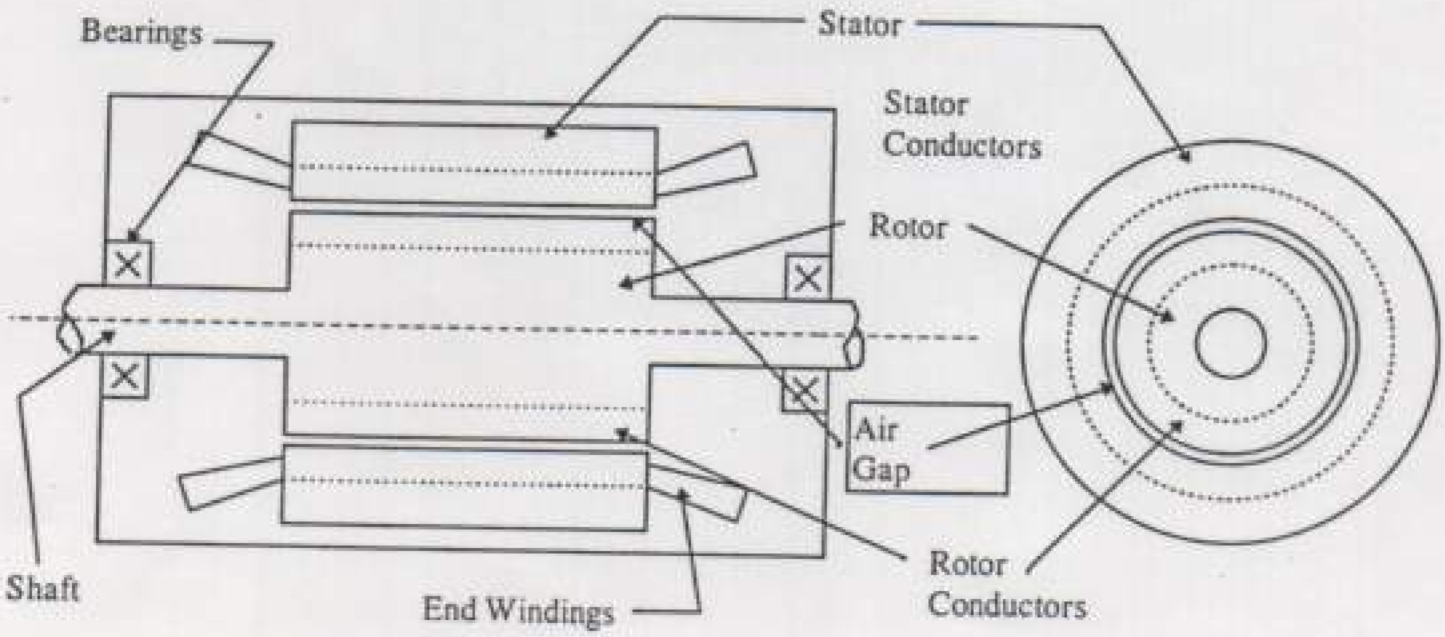

Figure 3: Form of Electric Machine

Consider the simplified machine drawing shown in Figure 3. Most (but not all!) machines we will be studying have essentially this morphology. The rotor of the machine is mounted on a shaft which is supported on some sort of bearing(s). Usually, but not always, the rotor is inside. I have drawn a rotor which is round, but this does not need to be the case. I have also indicated rotor conductors, but sometimes the rotor has permanent magnets either fastened to it or inside, and sometimes (as in Variable Reluctance Machines) it is just an oddly shaped piece of steel. The stator is, in this drawing, on the outside and has windings. With most of the machines we will be dealing with, the stator winding is the armature, or electrical power input element. (In DC and Universal motors this is reversed, with the armature contained on the rotor: we will deal with these later).

In most electrical machines the rotor and the stator are made of highly magnetically permeable materials: steel or magnetic iron. In many common machines such as induction motors the rotor and stator are both made up of thin sheets of silicon steel. Punched into those sheets are slots which contain the rotor and stator conductors.

Figure 4 is a picture of part of an induction machine distorted so that the air-gap is straightened out (as if the machine had infinite radius). This is actually a convenient way of drawing the machine and, we will find, leads to useful methods of analysis.

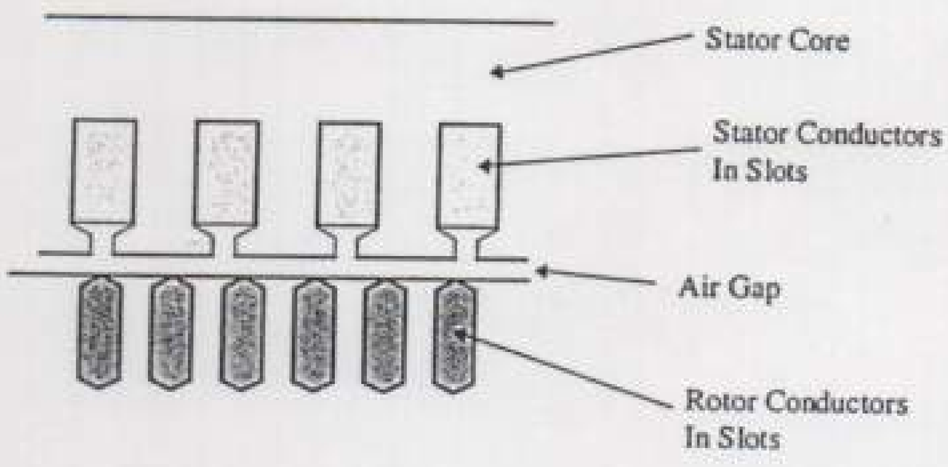

Figure 4: Windings in Slots

What is important to note for now is that the machine has an air gap g which is relatively small (that is, the gap dimension is much less than the machine radius r). The air-gap also has a physical length l. The electric machine works by producing a shear stress in the air-gap (with of course side effects such as production of "back voltage"). It is possible to define the average air-gap shear stress, which we will refer to as τ. Total developed torque is force over the surface area times moment (which is rotor radius):

$$T = 2\pi r^2 \ell < \tau >$$

Power transferred by this device is just torque times speed, which is the same as force times surface velocity, since surface velocity is $u = r\Omega$:

$$P_m = \Omega T = 2\pi r \ell < \tau > u$$

If we note that active rotor volume is , the ratio of torque to volume is just:

$$\frac{T}{V_r} = 2 < \tau >$$

Now, determining what can be done in a volume of machine involves two things. First, it is clear that the volume we have calculated here is not the whole machine volume, since it does not include the stator. The actual estimate of total machine volume from the rotor volume is actually quite complex and detailed and we will leave that one for later. Second, we need to estimate the value of the useful average shear stress. Suppose both the radial flux density Br and the stator surface current density Kz are sinusoidal flux waves of the form:

$$B_r = \sqrt{2} B_0 \cos(p\theta - \omega t)$$

$$K_z = \sqrt{2} K_0 \cos(p\theta - \omega t)$$

Note that this assumes these two quantities are exactly in phase, or oriented to ideally produce torque, so we are going to get an "optimistic" bound here. Then the average value of surface traction is:

$$< \tau > = \frac{1}{2\pi} \int_0^{2\pi} B_r K_z d\theta = B_0 K_0$$

This actually makes some sense in view of the empirically derived Lorentz Force Law: Given a (vector) current density and a (vector) flux density. In the absence of magnetic materials (those with permeability different from that of free space), the observed force on a conductor is:

$$\vec{F} = \vec{J} \times \vec{B}$$

Where $\vec{J}$ is the vector describing current density (A/m^2) and $\vec{B}$ is the magnetic flux density (T). This is actually enough to describe the forces we see in many machines, but since electric machines have permeable magnetic material and since magnetic fields produce forces on permeable material even in the absence of macroscopic currents it is necessary to observe how force appears on such material. A suitable empirical expression for force density is:

$$\vec{F} = \vec{J} \times \vec{B} - \frac{1}{2}\left(\vec{H} \cdot \vec{H}\right) \nabla \mu$$

where $\vec{H}$ is the magnetic field intensity and μ is the permeability.

Now, note that current density is the curl of magnetic field intensity, so that:

$$\begin{aligned} \vec{F} &= \left(\nabla \times \vec{H}\right) \times \mu \vec{H} - \frac{1}{2}\left(\vec{H} \cdot \vec{H}\right) \nabla \mu \\ &= \mu \left(\nabla \times \vec{H}\right) \times \vec{H} - \frac{1}{2}\left(\vec{H} \cdot \vec{H}\right) \nabla \mu \end{aligned}$$

And, since:

$$\left(\nabla \times \vec{H}\right) \times \vec{H} = \left(\vec{H} \cdot \nabla\right) \vec{H} - \frac{1}{2} \nabla \left(\vec{H} \cdot \vec{H}\right)$$

force density is:

$$\begin{aligned} \vec{F} &= \mu \left(\vec{H} \cdot \nabla\right) \vec{H} - \frac{1}{2} \mu \nabla \left(\vec{H} \cdot \vec{H}\right) - \frac{1}{2}\left(\vec{H} \cdot \vec{H}\right) \nabla \mu \\ &= \mu \left(\vec{H} \cdot \nabla\right) \vec{H} - \nabla \left(\frac{1}{2} \mu \left(\vec{H} \cdot \vec{H}\right)\right) \end{aligned}$$

This expression can be written by components: the component of force in the i'th dimension is:

$$F_i = \mu \sum_k \left(H_k \frac{\partial}{\partial x_k} \right) H_i - \frac{\partial}{\partial x_i} \left(\frac{1}{2} \mu \sum_k H_k^2 \right)$$

Now see that we can write the divergence of magnetic flux density as:

$$\nabla \cdot \vec{B} = \sum_k \frac{\partial}{\partial x_k} \mu H_k = 0$$

and

$$\mu \sum_k \left(H_k \frac{\partial}{\partial x_k} \right) H_i = \sum_k \frac{\partial}{\partial x_k} \mu H_k H_i - H_i \sum_k \frac{\partial}{\partial x_k} \mu H_k$$

but since the last term in that is zero, we can write force density as:

$$F_k = \frac{\partial}{\partial x_i} \left(\mu H_i H_k - \frac{\mu}{2} \delta_{ik} \sum_n H_n^2 \right)$$

where we have used the Kroneker delta $\delta_{ik} = 1$ if $i = k$, 0 otherwise.
Note that this force density is in the form of the divergence of a tensor:

$$F_k = \frac{\partial}{\partial x_i} T_{ik}$$

or

$$\vec{F} = \nabla \cdot \underline{\underline{T}}$$

In this case, force on some object that can be surrounded by a closed surface can be found by using the divergence theorem:

$$\vec{f} = \int_{\text{vol}} \vec{F} dv = \int_{\text{vol}} \nabla \cdot \underline{\underline{T}} dv = \oint\!\!\oint \underline{\underline{T}} \cdot \vec{n} da$$

or, if we note surface traction to be $\tau_i = \sum_k T_{ik} n_k$, where n is the surface normal vector, then the total force in direction i is just:

$$\vec{f} = \oint_s \tau_i da = \oint \sum_k T_{ik} n_k da$$

The interpretation of all of this is less difficult than the notation suggests. This field description of forces gives us a simple picture of surface traction, the force per unit area on a surface. If we just integrate this traction over the area of some body we get the whole force on the body.

Note one more thing about this notation. Sometimes when subscripts are repeated as they are here the summation symbol is omitted. Thus we would write $\tau_i = \sum_k T_{ik} n_k = T_{ik} n_k$.

Now, if we go back to the case of a circular cylinder and are interested in torque, it is pretty clear that we can compute the circumferential force by noting that the normal vector to the cylinder is just the radial unit vector, and then the circumferential traction must simply be:

$$\tau_\theta = \mu_0 H_r H_\theta$$

Assuming that there are no fluxes inside the surface of the rotor, simply integrating this over the surface gives azimuthal force, and then multiplying by radius (moment arm) gives torque. The last step is to note that, if the rotor is made of highly permeable material, the azimuthal magnetic field is equal to surface current density.

3 Tying the MST and Poynting Approaches Together

Figure 5: Illustrative Region of Space

Now that the stage is set, consider energy flow and force transfer in a narrow region of space as illustrated by Figure 5. The upper and lower surfaces may support currents. Assume that all of the fields, electric and magnetic, are of the form of a traveling wave in the x- direction: $\mathrm{Re}\left\{e^{j(\omega t-kx)}\right\}$.

If we assume that form for the fields and also assume that there is no variation in the z- direction (equivalently, the problem is infinitely long in the z- direction), there can be no x- directed currents because the divergence of current is zero: $\nabla \cdot \vec{J} = 0$. In a magnetostatic system this is true of electric field $\vec{E}$ too. Thus we will assume that current is confined to the z- direction and to the two surfaces illustrated in Figure 5, and thus the only important fields are:

$$\begin{aligned} \vec{E} &= \vec{i}_z \mathrm{Re}\left\{\underline{E}_z e^{j(\omega t-kx)}\right\} \\ \vec{H} &= \vec{i}_x \mathrm{Re}\left\{\underline{H}_x e^{j(\omega t-kx)}\right\} \\ &+ \vec{i}_y \mathrm{Re}\left\{\underline{H}_y e^{j(\omega t-kx)}\right\} \end{aligned}$$

We may use Faraday's Law ($\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$) to establish the relationship between the electric and magnetic field: the y- component of Faraday's Law is:

$$jk\underline{E}_z = -j\omega\mu_0 \underline{H}_y$$

or

$$\underline{E}_z = -\frac{\omega}{k}\mu_0 \underline{H}_y$$

The phase velocity $u_{ph} = \frac{\omega}{k}$ is a most important quantity. Note that, if one of the surfaces is moving (as it would be in, say, an induction machine), the frequency and hence the apparent phase velocity, will be shifted by the motion. We will use this fact shortly.

Energy flow through the surface denoted by the dotted line in Figure 5 is the component of Poynting's Vector in the negative y- direction. The relevant component is:

$$S_y = \left(\vec{E} \times \vec{H}\right)_y = E_z H_x = -\frac{\omega}{k}\mu_0 H_y H_x$$

Note that this expression contains the xy component of the Maxwell Stress Tensor $T_{xy} = \mu_0 H_x H_y$ so that power flow downward through the surface is:

$$\mathrm{S} = -S_y = \frac{\omega}{k}\mu_0 H_x H_y = u_{ph} T_{xy}$$

The *average* power flow is the same, in this case, for time and for space, and is:

$$< \mathrm{S} >= \frac{1}{2}\mathrm{Re}\left\{\underline{E}_z\underline{H}_x^*\right\} = u_{ph}\frac{\mu_0}{2}\mathrm{Re}\left\{\underline{H}_y\underline{H}_x^*\right\}$$

We may choose to define a *surface* impedance:

$$\underline{Z}_s = \frac{\underline{E}_z}{-\underline{H}_x}$$

which becomes:

$$\underline{Z}_s = -\mu_0 u_{ph}\frac{\underline{H}_y}{\underline{H}_x} = -\mu_0 u_{ph}\underline{R}$$

where now we have defined the parameter $\underline{R}$ to be the ratio between y- and x- directed complex field amplitudes. Energy flow through that surface is now:

$$\mathrm{S} = -\frac{1}{s}\mathrm{Re}\left\{\underline{E}_z\underline{H}_x^*\right\} = \frac{1}{2}\mathrm{Re}\left\{|\underline{H}_x|^2\underline{Z}_s\right\}$$

4 Simple Description of a Linear Induction Motor

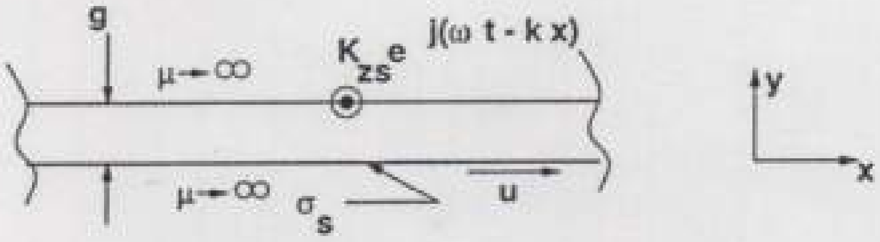

Figure 6: Simple Description of Linear Induction Motor

The stage is now set for an almost trivial description of a linear induction motor. Consider the geometry described in Figure 6. Shown here is *only* the relative motion gap region. This is bounded by two regions of highly permeable material (e.g. iron), comprising the stator and shuttle. On the surface of the stator (the upper region) is a surface current:

$$\vec{K}_s = \vec{i}_z\mathrm{Re}\left\{\underline{K}_{zs}e^{j(\omega t - kx)}\right\}$$

The shuttle is, in this case, moving in the positive x- direction at some velocity u. It may also be described as an infinitely permeable region with the capability of supporting a surface current with surface conductivity σ_s, so that $K_{zr} = \sigma_s E_z$.

Note that Ampere's Law gives us a boundary condition on magnetic field just below the upper surface of this problem: $H_x = K_{zs}$, so that, if we can establish the ratio between y- and x- directed fields at that location,

$$< T_{xy} >= \frac{\mu_0}{2}\mathrm{Re}\left\{\underline{H}_y\underline{H}_x^*\right\} = \frac{\mu_0}{2}|\underline{K}_{zs}|^2\mathrm{Re}\left\{\underline{R}\right\}$$

Note that the ratio of fields $\underline{H}_y/\underline{H}_x = \underline{R}$ is independent of reference frame (it doesn't matter if we are looking at the fields from the shuttle or the stator), so that the shear stress described by

T_{xy} is also frame independent. Now, if the shuttle (lower surface) is moving relative to the upper surface, the velocity of the traveling wave *relative to the shuttle* is:

$$u_s = u_{ph} - u = s\frac{\omega}{k}$$

where we have now defined the dimensionless *slip* s to be the ratio between frequency seen by the shuttle to frequency seen by the stator. We may use this to describe energy flow as described by Poynting's Theorem. Energy flow in the stator frame is:

$$S_{\text{upper}} = u_{ph}T_{xy}$$

In the frame of the shuttle, however, it is

$$S_{\text{lower}} = u_s T_{xy} = s S_{\text{upper}}$$

Now, the interpretation of this is that energy flow out of the upper surface (S_{upper}) consists of energy *converted* (mechanical power) plus energy dissipated in the shuttle (which is S_{lower} here. The difference between these two power flows, calculated using Poynting's Theorem, is power converted from electrical to mechanical form:

$$S_{\text{converted}} = S_{\text{upper}}(1-s)$$

Now, to finish the problem, note that surface current in the shuttle is:

$$\underline{K}_{zr} = \underline{E}'_z\sigma_s = -u_s\mu_0\sigma_s\underline{H}_y$$

where the electric field $\underline{E}'_z$ is measured in the frame of the shuttle.

We assume here that the magnetic gap g is small enough that we may assume $kg \ll 1$. Ampere's Law, taken around a contour that crosses the air-gap and has a normal in the z- direction, yields:

$$g\frac{\partial H_x}{\partial x} = K_{zs} + K_{zr}$$

In complex amplitudes, this is:

$$-jkg\underline{H}_y = \underline{K}_{zs} + \underline{K}_{zr} = \underline{K}_{zs} - \mu_0 u_s\sigma_s\underline{H}_y$$

or, solving for H_y.

$$\underline{H}_y = \frac{j\underline{K}_{zs}}{kg}\frac{1}{1+j\mu_0\frac{u_s\sigma_s}{kg}}$$

Average shear stress is

$$<T_{xy}>= \frac{\mu_0}{2}\text{Re}\left\{\underline{H}_y\underline{H}_x\right\} = \frac{\mu_0}{2}\frac{|\underline{K}_{zs}|^2}{kg}\text{Re}\left\{\frac{j}{1+j\frac{\mu_0 u_s\sigma_s}{kg}}\right\} = \frac{\mu_0}{2}\frac{|\underline{K}_{zs}|^2}{kg}\frac{\frac{\mu_0 u_s\sigma_s}{kg}}{1+\left(\frac{\mu_0 u_s\sigma_s}{kg}\right)^2}$$

A good exercise in MATLAB would be to evaluate this expression, either for some reasonably normalized set of parameters or for realistic numbers.

5 Surface Impedance of Uniform Conductors

The objective of this section is to describe the calculation of the surface impedance presented by a layer of conductive material. Two problems are considered here. The first considers a layer of *linear* material backed up by an infinitely permeable surface. This is approximately the situation presented by, for example, surface mounted permanent magnets and is probably a decent approximation to the conduction mechanism that would be responsible for loss due to asynchronous harmonics in these machines. It is also appropriate for use in estimating losses in solid rotor induction machines and in the poles of turbogenerators. The second problem, which we do not work here but simply present the previously worked solution, concerns saturating ferromagnetic material.

5.1 Linear Case

The situation and coordinate system are shown in Figure 7. The conductive layer is of thicknes T and has conductivity σ and permeability μ_0. To keep the mathematical expressions within bounds, we assume rectilinear geometry. This assumption will present errors which are small to the extent that curvature of the problem is small compared with the wavenumbers encountered. We presume that the situation is excited, as it would be in an electric machine, by a current sheet of the form $K_z = Re\left\{\underline{K}e^{j(\omega t - kx)}\right\}$

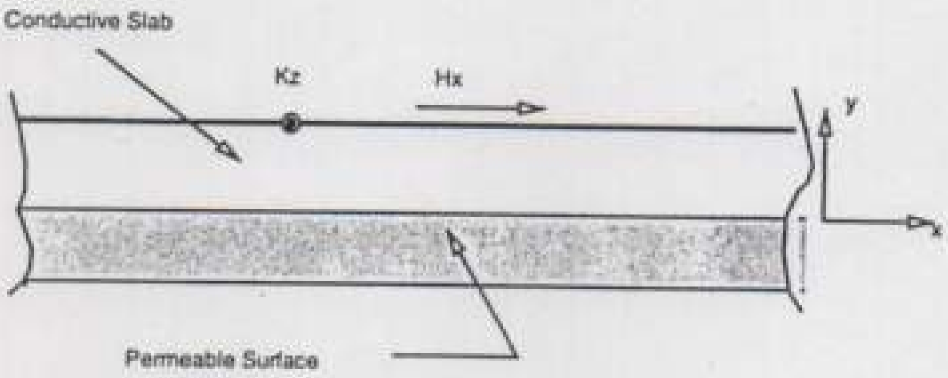

Figure 7: Axial View of Magnetic Field Problem

In the conducting material, we must satisfy the diffusion equation:

$$\nabla^2 \overline{H} = \mu_0 \sigma \frac{\partial \overline{H}}{\partial t}$$

In view of the boundary condition at the back surface of the material, taking that point to be $y = 0$, a general solution for the magnetic field in the material is:

$$\begin{aligned} H_x &= Re\left\{A \sinh \alpha y e^{j(\omega t - kx)}\right\} \\ H_y &= Re\left\{j\frac{k}{\alpha} A \cosh \alpha y e^{j(\omega t - kx)}\right\} \end{aligned}$$

where the coefficient α satisfies:

$$\alpha^2 = j\omega\mu_0\sigma + k^2$$

and note that the coefficients above are chosen so that $\overline{H}$ has no divergence.

Note that if k is small (that is, if the wavelength of the excitation is large), this spatial coefficient α becomes

$$\alpha = \frac{1+j}{\delta}$$

where ~~the~~ skin depth is:

$$\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}}$$

Faraday's law:

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

gives:

$$\underline{E}_z = -\mu_0 \frac{\omega}{k} \underline{H}_y$$

Now: the "surface current" is just

$$\underline{K}_s = -\underline{H}_x$$

so that the equivalent surface impedance is:

$$\underline{Z} = \frac{\underline{E}_z}{-\underline{H}_x} = j\mu_0 \frac{\omega}{\alpha} \coth \alpha T$$

A pair of limits are interesting here. Assuming that the wavelength is long so that k is negligible, then if αT is *small* (i.e. thin material),

$$\underline{Z} \to j\mu_0 \frac{\omega}{\alpha^2 T} = \frac{1}{\sigma T}$$

On the other hand as $\alpha T \to \infty$,

$$\underline{Z} \to \frac{1+j}{\sigma \delta}$$

Next it is necessary to transfer this surface impedance across the air-gap of a machine. So, assume a new coordinate system in which the surface of impedance $\underline{Z}_s$ is located at $y = 0$, and we wish to determine the impedance $\underline{Z} = -\underline{E}_z / \underline{H}_x$ at $y = g$.

In the gap there is no current, so magnetic field can be expressed as the gradient of a scalar potential which obeys Laplace's equation:

$$\overline{H} = -\nabla \psi$$

and

$$\nabla^2 \psi = 0$$

Ignoring a common factor of $e^{j(\omega t - kx)}$, we can express $\overline{H}$ in the gap as:

$$\begin{aligned} \underline{H}_x &= jk \left(\underline{\psi}_+ e^{ky} + \underline{\psi}_- e^{-ky} \right) \\ \underline{H}_y &= -k \left(\underline{\psi}_+ e^{ky} - \underline{\psi}_- e^{-ky} \right) \end{aligned}$$

At the surface of the rotor,

$$\underline{E}_z = -\underline{H}_x \underline{Z}_s$$

or

$$-\omega\mu_0\left(\underline{\psi}_+ - \underline{\psi}_-\right) = jk\underline{Z}_s\left(\underline{\psi}_+ + \underline{\psi}_-\right)$$

and then, at the surface of the stator,

$$\underline{Z} = -\frac{\underline{E}_z}{\underline{H}_x} = j\mu_0\frac{\omega}{k}\frac{\underline{\psi}_+ e^{kg} - \underline{\psi}_- e^{-kg}}{\underline{\psi}_+ e^{kg} + \underline{\psi}_- e^{-kg}}$$

A bit of manipulation is required to obtain:

$$\underline{Z} = j\mu_0\frac{\omega}{k}\left\{\frac{e^{kg}\left(\omega\mu_0 - jk\underline{Z}_s\right) - e^{-kg}\left(\omega\mu_0 + jk\underline{Z}_s\right)}{e^{kg}\left(\omega\mu_0 - jk\underline{Z}_s\right) + e^{-kg}\left(\omega\mu_0 + jk\underline{Z}_s\right)}\right\}$$

It is useful to note that, in the limit of $\underline{Z}_s \rightarrow \infty$, this expression approaches the *gap impedance*

$$\underline{Z}_g = j\frac{\omega\mu_0}{k^2 g}$$

and, if the gap is small enough that $kg \rightarrow 0$,

$$\underline{Z} \rightarrow \underline{Z}_g || \underline{Z}_s$$

6 Iron

Electric machines employ ferromagnetic materials to carry magnetic flux from and to appropriate places within the machine. Such materials have properties which are interesting, useful and problematical, and the designers of electric machines must deal with this stuff. The purpose of this note is to introduce the most salient properties of the kinds of magnetic materials used in electric machines.

We will be concerned here with materials which exhibit *magnetization*: flux density is something other than $\vec{B} = \mu_0\vec{H}$. Generally, we will speak of *hard* and *soft* magnetic materials. Hard materials are those in which the magnetization tends to be permanent, while soft materials are used in magnetic circuits of electric machines and transformers. Since they are related we will find ourselves talking about them either at the same time or in close proximity, even though their uses are widely disparite.

6.1 Magnetization:

It is possible to relate, in all materials, magnetic flux density to magnetic field intensity with a consitutive relationship of the form:

$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)$$

where magnetic field intensity H and magnetization M are the two important properties. Now, in linear magnetic material magnetization is a simple linear function of magnetic field:

$$\vec{M} = \chi_m\vec{H}$$

so that the flux density is also a linear function:

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H}$$

Note that in the most general case the magnetic susceptibility cm might be a tensor, leading to flux density being non-colinear with magnetic field intensity. But such a relationship would still be linear. Generally this sort of complexity does not have a major effect on electric machines.

6.2 Saturation and Hysteresis

In useful magnetic materials this nice relationship is not correct and we need to take a more general view. We will not deal with the microscopic picture here, except to note that the magnetization is due to the alignment of groups of magnetic dipoles, the groups often called *domaines*. There are only so many magnetic dipoles available in any given material, so that once the flux density is high enough the material is said to saturate, and the relationship between magnetic flux density and magnetic field intensity is nonlinear.

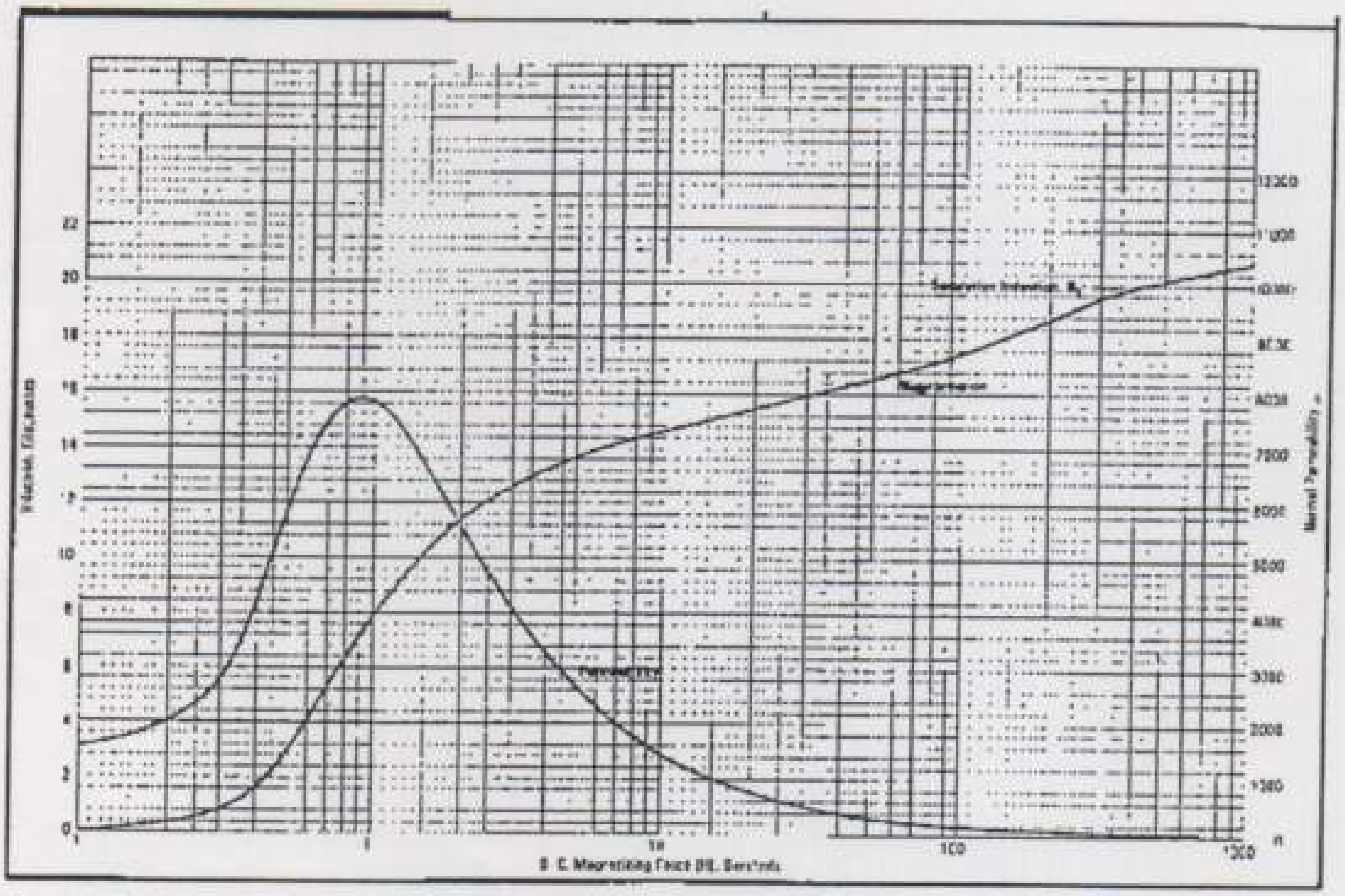

Figure 8: Saturation Curve: Commercial M-19 Silicon Iron

Shown in Figure 8, for example, is a "saturation curve" for a magnetic sheet steel that is sometimes used in electric machinery. Note the magnetic field intensity is on a logarithmic scale. If this were plotted on linear coordinates the saturation would appear to be quite abrupt.

At this point it is appropriate to note that the units used in magnetic field analysis are not always the same nor even consistent. In almost all systems the unit of flux is the weber (W), which is the same as a volt-second. In SI the unit of flux density is the tesla (T), but many people refer to the gauss (G), which has its origin in CGS. 10,000 G = 1 T. Now it gets worse, because there is an English system measure of flux density generally called kilo-lines per square inch. This is because in the English system the unit of flux is the line. 10^8 lines is equal to a weber. Thus a Tesla is 64.5 kilolines per square inch.

The SI and CGS units of flux density are easy to reconcile, but the units of magnetic field are a bit harder. In SI we generally measure H in amperes/meter (or ampere-turns per meter). Often, however, you will see magnetic field represented as Oersteds (Oe). One Oe is the same as the magnetic field required to produce one gauss in free space. So 79.577 A/m is one Oe.

In most useful magnetic materials the magnetic domaines tend to be somewhat "sticky", and a more-than-incremental magnetic field is required to get them to move. This leads to the property called "hysteresis", both useful and problematical in many magnetic systems.

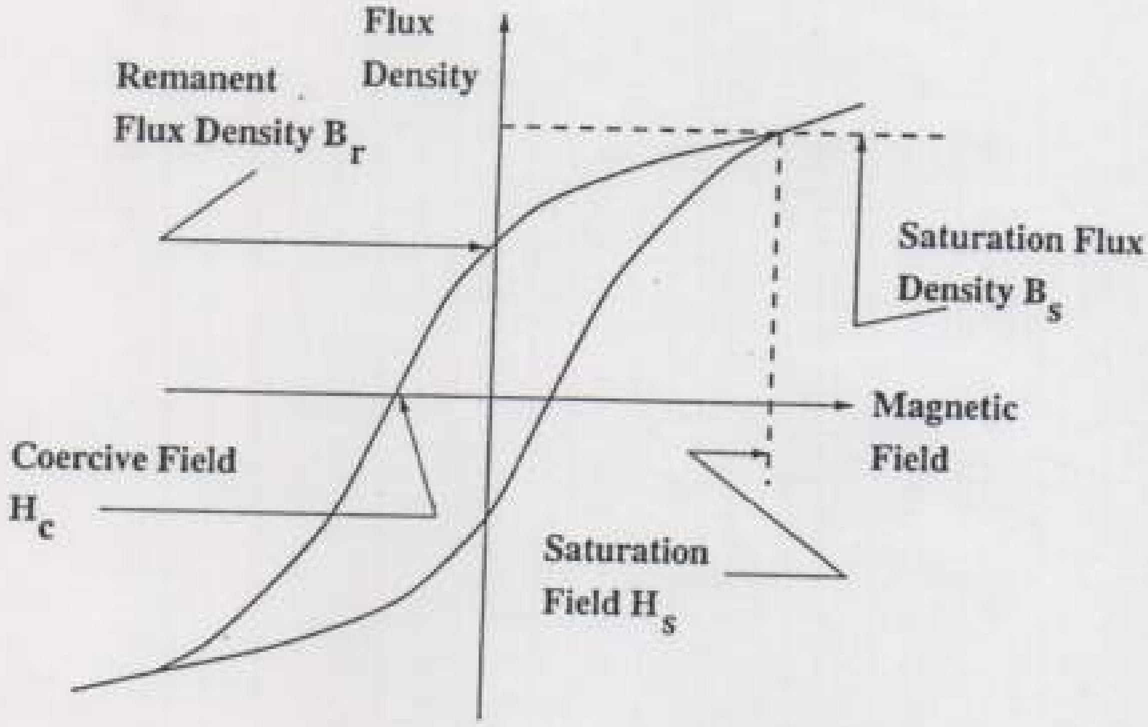

Figure 9: Hysteresis Curve Nomenclature

Hysteresis loops take many forms; a generalized picture of one is shown in Figure 9. Salient features of the hysteresis curve are the remanent magnetization B_r and the coercive field H_c. Note that the actual loop that will be traced out is a function of field amplitude and history. Thus there are many other "minor loops" that might be traced out by the B-H characteristic of a piece of material, depending on just what the fields and fluxes have done and are doing.

Now, hysteresis is important for two reasons. First, it represents the mechanism for "trapping" magnetic flux in a piece of material to form a permanent magnet. We will have more to say about that anon. Second, hysteresis is a loss mechanism. To show this, consider some arbitrary chunk of material for which we can characterize an MMF and a flux:

$$F = NI = \int \vec{H} \cdot d\vec{ell}$$
$$\Phi = \int \frac{V}{N} dt = \iint_{\text{Area}} \vec{B} \cdot d\vec{A}$$

Energy input to the chunk of material over some period of time is

$$w = \int VIdt = \int Fd\Phi = \int_t \int \vec{H} \cdot d\vec{\ell} \iint d\vec{B} \cdot d\vec{A}\, dt$$

Now, imagine carrying out the second (double) integral over a continuous set of surfaces which are perpendicular to the magnetic field H. (This IS possible!). The energy becomes:

$$w = \int_t \iiint \vec{H} \cdot d\vec{B} \mathrm{dvol}\, dt$$

and, done over a complete cycle of some input waveform, that is:

$$w = \iiint_{\mathrm{vol}} W_m \mathrm{dvol}$$
$$W_m = \oint_t \vec{H} \cdot d\vec{B}$$

That last expression simply expresses the area of the hysteresis loop for the particular cycle.

Generally, for most electric machine applications we will use magnetic material characterized as "soft", having as narrow a hysteresis loop (and therefore as low a hysteretic loss) as possible. At the other end of the spectrum are "hard" magnetic materials which are used to make permanent magnets. The terminology comes from steel, in which soft, annealed steel material tends to have narrow loops and hardened steel tends to have wider loops. However permanent magnet technology has advanced to the point where the coercive forces possible in even cheap ceramic magnets far exceed those of the hardest steels.

6.3 Conduction, Eddy Currents and Laminations:

Steel, being a metal, is an electrical conductor. Thus when time varying magnetic fields pass through it they cause eddy currents to flow, and of course those produce dissipation. In fact, for almost all applications involving "soft" iron, eddy currents are the dominant source of loss. To reduce the eddy current loss, magnetic circuits of transformers and electric machines are almost invariably laminated, or made up of relatively thin sheets of steel. To further reduce losses the steel is alloyed with elements (often silicon) which poison the electrical conductivity.

There are several approaches to estimating the loss due to eddy currents in steel sheets and in the surface of solid iron, and it is worthwhile to look at a few of them. It should be noted that this is a "hard" problem, since the behavior of the material itself is difficult to characterize.

6.4 Complete Penetration Case

Consider the problem of a stack of laminations. In particular, consider one sheet in the stack represented in Figure 10. It has thickness t and conductivity σ. Assume that the "skin depth" is much greater than the sheet thickness so that magnetic field penetrates the sheet completely. Further, assume that the applied magnetic flux density is parallel to the surface of the sheets:

$$\vec{B} = \vec{i}_z \mathrm{Re}\left\{\sqrt{2} B_0 e^{j\omega t}\right\}$$

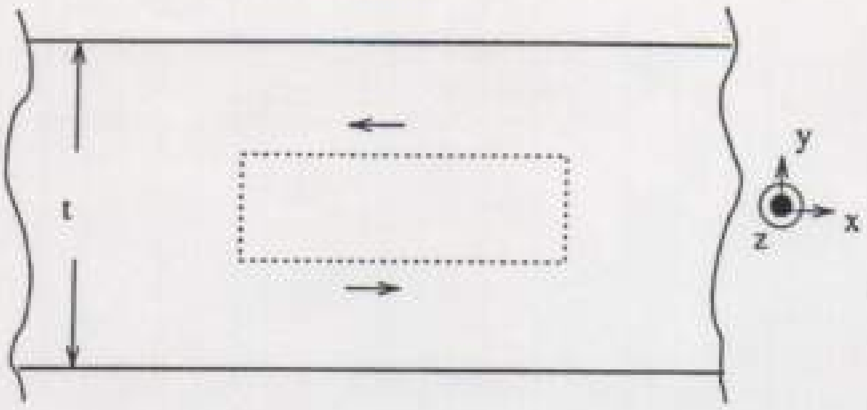

Figure 10: Lamination Section for Loss Calculation

Now we can use Faraday's law to determine the electric field and therefore current density in the sheet. If the problem is uniform in the x- and z- directions,

$$\frac{\partial \underline{E}_x}{\partial y} = -j\omega_0 B_0$$

Note also that, unless there is some net transport current in the x- direction, E must be anti-symmetric about the center of the sheet. Thus if we take the origin of y to be in the center, electric field and current are:

$$\begin{aligned} \underline{E}_x &= -j\omega B_0 y \\ \underline{J}_x &= -j\omega B_0 \sigma y \end{aligned}$$

Local power dissipated is

$$P(y) = \omega^2 B_0^2 \sigma y^2 = \frac{|J|^2}{\sigma}$$

To find average power dissipated we integrate over the thickness of the lamination:

$$< P >= \frac{2}{t}\int_0^{\frac{t}{2}} P(y)dy = \frac{2}{t}\omega^2 B_0^2 \sigma \int_0^{\frac{t}{2}} y^2 dy = \frac{1}{12}\omega^2 B_0^2 t^2 \sigma$$

Pay attention to the orders of the various terms here: power is proportional to the square of flux density and to the square of frequency. It is also proportional to the square of the lamination thickness (this is average volume power dissipation).

As an aside, consider a simple magnetic circuit made of this material, with some length ℓ and area A, so that volume of material is ℓA. Flux lined by a coil of N turns would be:

$$\Lambda = N\Phi = NAB_0$$

and voltage is of course just $V = jwL$. Total power dissipated in this core would be:

$$P_c = A\ell \frac{1}{12}\omega^2 B_0^2 t^2 \sigma = \frac{V^2}{R_c}$$

where the equivalent core resistance is now

$$R_c = \frac{A}{\ell}\frac{12N^2}{\sigma t^2}$$

6.5 Eddy Currents in Saturating Iron

The same geometry holds for this pattern, although we consider only the one-dimensional problem ($k \rightarrow 0$). The problem was worked by McLean and his graduate student Agarwal [2] [1]. They assumed that the magnetic field at the surface of the flat slab of material was sinusoidal in time and of high enough amplitude to saturate the material. This is true if the material has high permeability and the magnetic field is strong. What happens is that the impressed magnetic field saturates a region of material near the surface, leading to a magnetic flux density parallel to the surface. The depth of the region affected changes with time, and there is a separating surface (in the flat problem this is a plane) that moves away from the top surface in response to the change in the magnetic field. An electric field is developed to move the surface, and that magnetic field drives eddy currents in the material.

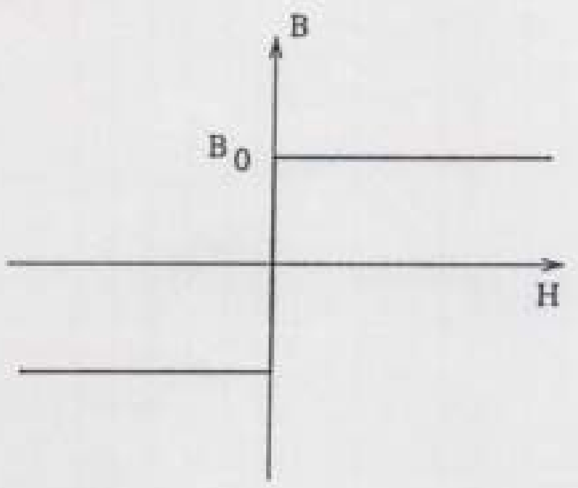

Figure 11: Idealized Saturating Characteristic

Assume that the material has a perfectly rectangular magnetization curve as shown in Figure 11, so that flux density in the x- direction is:

$$B_x = B_0 \text{sign}(H_x)$$

The flux per unit width (in the z- direction) is:

$$\Phi = \int_0^{-\infty} B_x dy$$

and Faraday's law becomes:

$$E_z = \frac{\partial \Phi}{\partial t}$$

while Ampere's law in conjunction with Ohm's law is:

$$\frac{\partial H_x}{\partial y} = \sigma E_z$$

Now, McLean suggested a solution to this set in which there is a "separating surface" at depth ζ below the surface, as shown in Figure 12 . At any given time:

$$\begin{aligned} H_x &= H_s(t)\left(1 + \frac{y}{\zeta}\right) \\ J_z &= \sigma E_z = \frac{H_s}{\zeta} \end{aligned}$$

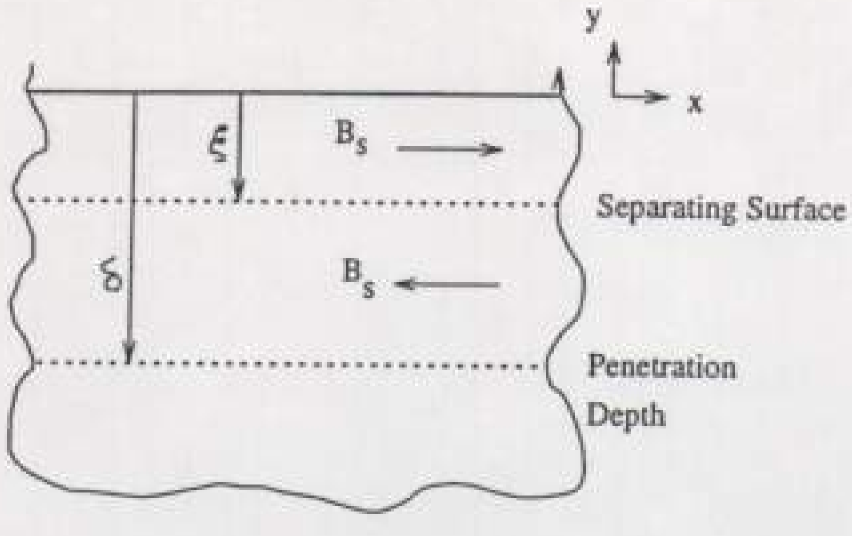

Figure 12: Separating Surface and Penetration Depth

That is, in the region between the separating surface and the top of the material, electric field E_z is uniform and magnetic field H_x is a linear function of depth, falling from its impressed value at the surface to zero at the separating surface. Now: electric field is produced by the rate of change of flux which is:

$$E_z = \frac{\partial \Phi}{\partial t} = 2B_x \frac{\partial \zeta}{\partial t}$$

Eliminating E, we have:

$$2\zeta \frac{\partial \zeta}{\partial t} = \frac{H_s}{\sigma B_x}$$

and then, if the impressed magnetic field is sinusoidal, this becomes:

$$\frac{d\zeta^2}{dt} = \frac{H_0}{\sigma B_0} |\sin \omega t|$$

This is easy to solve, assuming that $\zeta = 0$ at $t = 0$,

$$\zeta = \sqrt{\frac{2H_0}{\omega \sigma B_0}} \sin \frac{\omega t}{2}$$

Now: the surface always moves in the downward direction (as we have drawn it), so at each half cycle a new surface is created: the old one just stops moving at a maximum position, or penetration depth:

$$\delta = \sqrt{\frac{2H_0}{\omega \sigma B_0}}$$

This penetration depth is analogous to the "skin depth" of the linear theory. However, it is an absolute penetration depth.

The resulting electric field is:

$$E_z = \frac{2H_0}{\sigma \delta} \cos \frac{\omega t}{2} \qquad 0 < \omega t < \pi$$

This may be Fourier analyzed: noting that if the impressed magnetic field is sinusoidal, only the time fundamental component of electric field is important, leading to:

$$E_z = \frac{8}{3\pi} \frac{H_0}{\sigma \delta} (\cos \omega t + 2 \sin \omega t + \ldots)$$

Complex surface impedance is the ratio between the complex amplitude of electric and magnetic field, which becomes:

$$\underline{Z}_s = \frac{\underline{E}_z}{\underline{H}_x} = \frac{8}{3\pi}\frac{1}{\sigma\delta}(2+j)$$

Thus, in practical applications, we can handle this surface much as we handle linear conductive surfaces, by establishing a skin depth and assuming that current flows within that skin depth of the surface. The resistance is modified by the factor of $\frac{16}{3\pi}$ and the "power factor" of this surface is about 89 % (as opposed to a linear surface where the "power factor" is about 71 %.

Agarwal suggests using a value for B_0 of about 75 % of the saturation flux density of the steel.

7 Semi-Empirical Method of Handling Iron Loss

Neither of the models described so far are fully satisfactory in describing the behavior of laminated iron, because losses are a combination of eddy current and hysteresis losses. The rather simple model employed for eddy currents is precise because of its assumption of abrupt saturation. The hysteresis model, while precise, would require an empirical determination of the size of the hysteresis loops anyway. So we must often resort to empirical loss data. Manufacturers of lamination steel sheets will publish data, usually in the form of curves, for many of their products. Here are a few ways of looking at the data.

A low frequency flux density vs. magnetic field ("saturation") curve was shown in Figure 8. Included with that was a measure of the incremental permeability

$$\mu' = \frac{dB}{dH}$$

In *some* machine applications either the "total" inductance (ratio of flux to MMF) or "incremental" inductance (slope of the flux to MMF curve) is required. In the limit of low frequency these numbers may be useful.

For designing electric machines, however, a second way of looking at steel may be more useful. This is to measure the real and reactive power as a function of magnetic flux density and (sometimes) frequency. In principal, this data is immediately useful. In any well-designed electric machine the flux density in the core is distributed fairly uniformly and is not strongly affected by eddy currents, etc. in the core. Under such circumstances one can determine the flux density in each part of the core. With that information one can go to the published empirical data for real and reactive power and determine core loss and reactive power requirements.

Figure 13 shows core loss and "apparent" power per unit mass as a function of (RMS) induction for 29 gage, fully processed M-19 steel. The two left-hand curves are the ones we will find most useful. "P" denotes real power while "P_a" denotes "apparent power". The use of this data is quite straightforward. If the flux density in a machine is estimated for each part of the machine and the mass of steel calculated, then with the help of this chart a total core loss and apparent power can be estimated. Then the effect of the core may be approximated with a pair of elements in parallel with the terminals, with:

$$R_c = \frac{q|V|^2}{P}$$
$$X_c = \frac{q|V|^2}{Q}$$

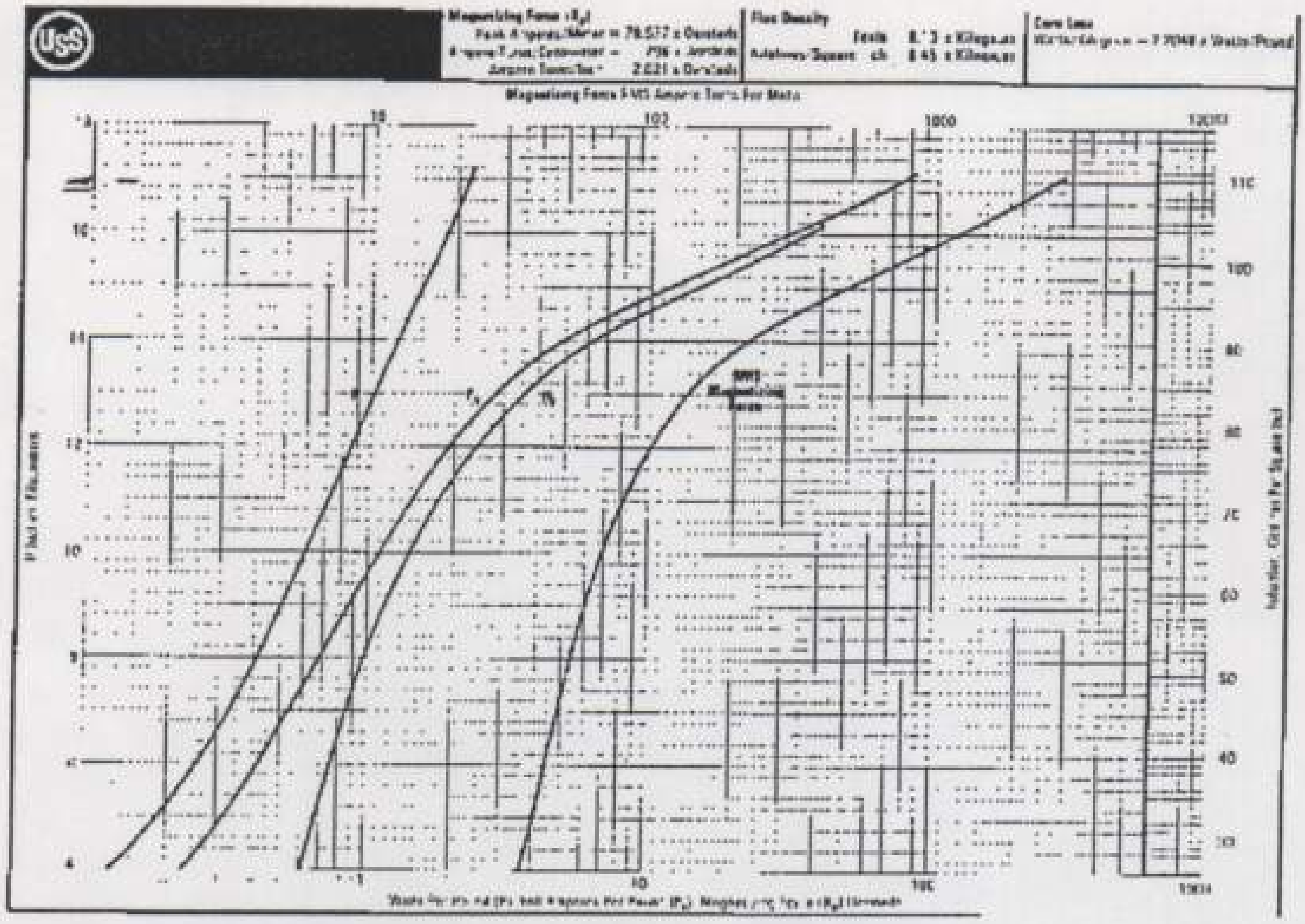

Figure 13: Real and Apparent Loss: M19, Fully Processed, 29 Ga

$$Q = \sqrt{P_a^2 - P^2}$$

Where q is the number of machine phases and V is *phase* voltage. Note that this picture is, strictly speaking, only valid for the voltage and frequency for which the flux density was calculated. But it will be approximately true for small excursions in either voltage or frequency and therefore useful for estimating voltage drop due to exciting current and such matters. In design program applications these parameters can be re-calculated repeatedly if necessary.

"Looking up" this data is a it awkward for design studies, so it is often convenient to do a "curve fit" to the published data. There are a large number of possible ways of doing this. One method that has bee found to work reasonably well for silicon iron is an "exponential fit":

$$P \approx P_0 \left(\frac{B}{B_0}\right)^{\epsilon_B} \left(\frac{f}{f_0}\right)^{\epsilon_F}$$

This fit is appropriate if the data appears on a log-log plot to lie in approximately straight lines. Figure 14 shows such a fit for the same steel sheet as the other figures.

For "apparent power" the same sort of method can be used. It appears, however, that the simple exponential fit which works well for real power is inadequate, at least if relatively high inductions are to be used. This is because, as the steel saturates, the reactive component of exciting current rises rapidly. I have had some success with a "double exponential" fit:

$$VA \approx VA_0 \left(\frac{B}{B_0}\right)^{\epsilon_0} + VA_1 \left(\frac{B}{B_0}\right)^{\epsilon_1}$$

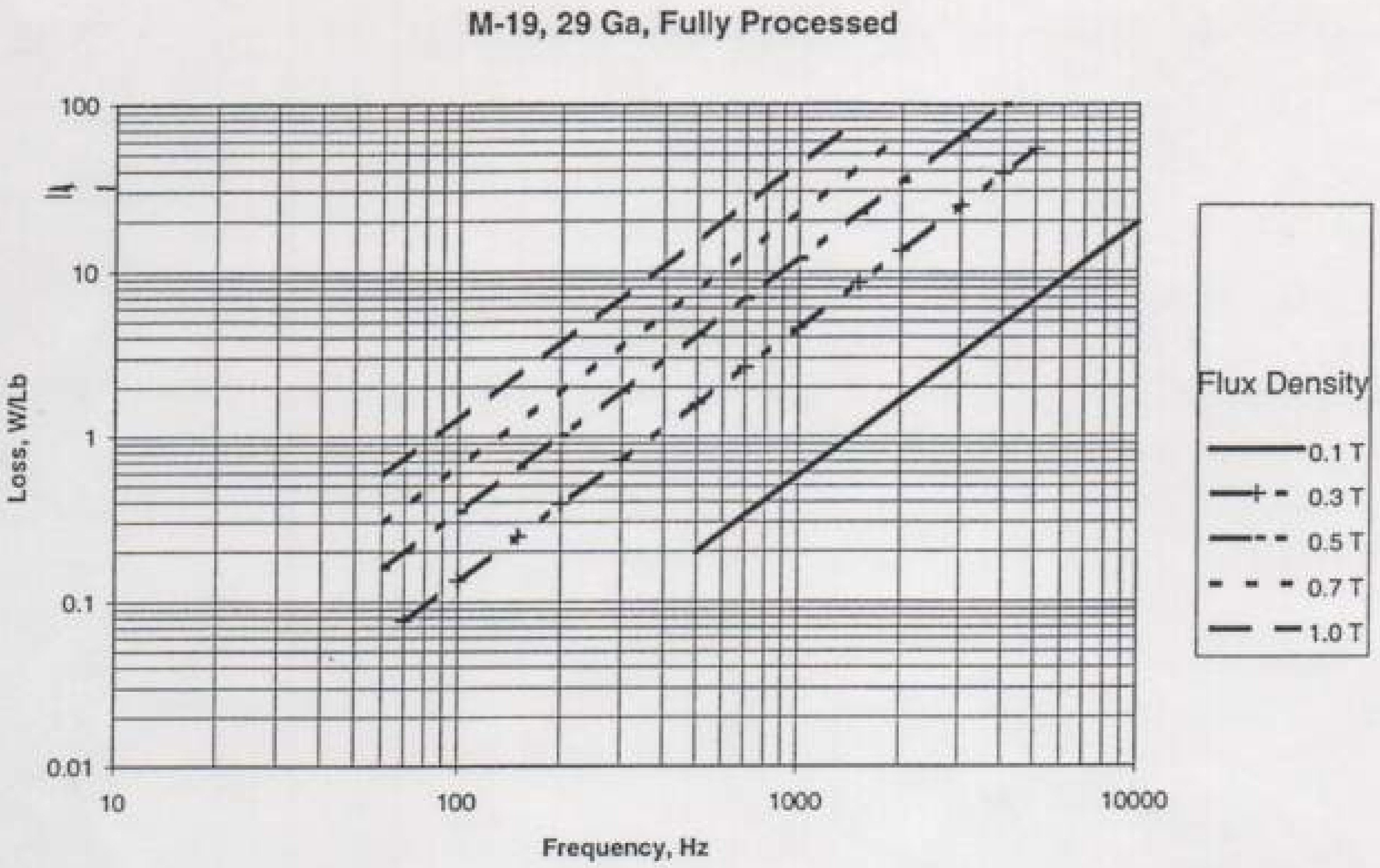

Figure 14: Steel Sheet Core Loss Fit vs. Flux Density and Frequency

To first order the reactive component of exciting current will be linear in frequency.

In the disk that is to be distributed with these notes there are a number of data files representing properties of different types of nonoriented sheet steel. The format of each of the files is the same: two columns of numbers, the first is flux density in Tesla, RMS, 60 Hz. The second column is watts per pound or volt-amperes per pound. The materials are denoted by the file names, which are generally of the format: "M-Mtype-Proc-Data-Gage.prn". The coding is relatively dense because of the short file name limit of MSDOS. Mtype is the number designator (as in M-19). Proc is "f" for fully processed and "s" for semiprocessed. Data is "p" for power, "pa" for apparent power. Gage is 29 (.014" thick), 26 (.0185" thick) or 24 (.025" thick). Example: `m19fp29.prn` designates loss in M-19 material, fully processed, 29 gage.

Also on the disk are three curve fitting routines that appear to work with this data. (Not all of the routines work with all of the data!). They are:

1. `efit.m` implements the single exponential fit of loss against flux density. Use: in MATLAB type

   ```
   efit <return>.
   ```

   The program prompts

   ```
   fit what (name.prn) ==>
   ```

   Enter the file name for the material designator without the .prn extension. The program will think about the problem for a few seconds and put up a plot of its fit with points

Table 1: Exponential Fit Parameters for Two Steel Sheets

		29 Ga, Fully Processed	
		M-19	M-36
Base Flux Density	B_0	1 T	1 T
Base Frequency	f_0	60 Hz	60 Hz
Base Power (w/lb)	P_0	0.59	0.67
Flux Exponent	ϵ_B	1.88	1.86
Frequency Exponent	ϵ_F	1.53	1.48
Base Apparent Power 1	VA_0	1.08	1.33
Base Apparent Power 2	VA_1	.0144	.0119
Flux Exponent	ϵ_0	1.70	2.01
Flux Exponent	ϵ_1	16.1	17.2

noting the actual data. Enter a `<return>` and a summary of the fit turns up, including the fit parameters and an error indication. These programs use MATLAB's `fmins` routine to minimize a mean-squared error as calculated by the auxiliary function `fiterr.m`.

2. `e2fit.m` implements the double exponential fit of apparent power against flux density. Use is just like efit. It uses the auxiliary function `fit2err.m`.

3. `pfit.m` uses the MATLAB function `polyfit` to fit a polynomial (in B) to the data.

Most of the machine design scripts enclosed with the material for this special summer subject employ the exponential fits for core iron developed here.

References

[1] W. MacLean, "Theory of Strong Electromagnetic Waves in Massive Iron", Journal of Applied Physics, V.25, No 10, October, 1954

[2] P.D. Agarwal, "Eddy-Current Losses in Solid and Laminated Iron", Trans. AIEE, V. 78, pp 169-171, 1959